

Fundamentals of Financial Arithmetics

Name of the course

Wioletta Nowak

The Professor

Biography of the Professor

Wioletta Nowak is an assistant professor at the Institute of Economic Sciences at the Faculty of Law, Administration and Economics of the University of Wrocław, Poland. She has a MSc in theoretical physics and a MA in philosophy from University of Wrocław, and PhD in economics from the former Higher School of Economics in Wrocław (Wrocław University of Economics today). Her research interests span mathematical economics, economic growth, economic development and international economics. She is (co)author of 5 books and over 60 papers.

Requirements for passing a course:

Written exam, five exercises based on exercises from the list below, to pass a student needs 50%, for B – 70% and 90% for A.

Exam exercises:

1. Calculate the amount of interest that will be paid on an investment of 6 000 PLN at 9% simple interest per annum for 6 years and 4 months.
2. A bank charges simple interest at a rate of 6% p.a. on a 90-day loan of 10 000 PLN. Calculate the interest.
3. Suppose you put 1 000 PLN in a savings account paying simple interest at 4% per annum for two years. Then, you withdraw the money with interest and put it for a half of year in another account paying simple interest at 5%. How much do you have in the end?
4. How many quarters does it take for 400 PLN to accumulate to 2 000 PLN under 8% p.a. simple interest?
5. Calculate the time taken for 2000 PLN to earn 50 PLN at 5% simple interest.
6. Find the simple interest rate if principal increases five times in twenty years.

7. Calculate the difference between the simple interest and the compound interest on a loan of 2 000 PLN at 7.5% per annum over 5 years.
8. Calculate the final amount if 4500 PLN is invested at 6% compound interest per annum for 10 years.
9. Suppose that you save 1 000 PLN in an account that pays 3% interest every quarter. How much do you have in one year? Compound interest is used.
10. Calculate the amount of compound interest earned from investing 8 000 PLN at 8% per annum for 3 years.
11. How much do you need to invest now to get 5000 PLN after two years if the interest rate is $5\frac{1}{2}\%$. Compound interest is used.
12. Suppose that 2 000 PLN is invested at 2.5% per annum, compounding monthly. How much interest is paid in the fourth year of investment?
13. How long does it take to double your capital if you put it in an account paying compound interest rate at a rate of $4\frac{3}{4}\%$? What if the account pays simple interest? What if the account pays continuously compounded interest?
14. Compute the nominal interest rate per annum payable monthly that is equivalent to the simple interest rate of 9% per annum over a period of nine months.
15. Calculate the final amount if 3 500 PLN is invested at 5% continuously compounded interest for 5 years.
16. What is the present value of 8 000 PLN to be received in 6 years using 9% continuous discounting?
17. Suppose that an account offers a nominal interest rate of 6% p.a. payable monthly. What is effective interest rate? What if nominal rate is the same, but interest is payable daily? Weekly? Quarterly?
18. Compare the following three loans: a loan charging 10% compounded monthly, a loan charging 11% compounded quarterly, a loan charging an annual effective rate of 10.5%.
19. An investor wishes to accumulate 10 000 PLN in a fund at the end of 4 years. To accomplish this the investor plans to make deposits at the beginning of each month. How much should be each deposit if the simple interest rate is 9% per annum. Rework example with deposits at the end of each month.
20. Consider an annuity-due paying 10 PLN at the beginning of every month for 6 years. The simple interest rate is 8% per annum. What an amount of money you should pay at the end of each year for 6 years to obtain the same future value if the simple interest rate is 12% per annum.
21. Find the accumulated value at the end of six months of a fund in which 10 PLN, 20 PLN, and 30 PLN are deposited respectively at the beginning of the first three months, and at the end of the next three months the same amounts and in the same order are withdrawn. The simple interest rate is 10% per annum.
22. At what of simple interest, the future value of annual annuity-due at the end of 3 years is twice the present value of annual annuity-immediate at the end of 5 years. The value of both types of payments is the same.
23. An investor wishes to accumulate 100 PLN. How many months must an investor pay monthly annuity-due of 8 PLN if the simple interest rate is 24% per annum. Solve the problem of non-integer value of the term of annuities.
24. If at the beginning of each year, an investor deposited 100 PLN into a savings account that paid 6%, compounding monthly, how much would investor have at the end of 10 years?
25. Find the present value of an annuity which pays 300 PLN at the end of each quarter for 10 years if the rate of interest is 9% convertible monthly (quarterly, semi-annually, annually).

26. Find the accumulated value at the end of five years of a fund if 5 PLN was deposited at the beginning of each month and 30 PLN was withdrawn at the end of each semi-year. The interest rate was 8% compounded quarterly.
27. An investor wishes to accumulate 100 PLN. How many years must the investor pay 20 PLN at the beginning of each year if the interest rate is 11% per annum, compounding annually. Solve the problem of non-integer value of the term of annuities.
28. Find the future value of an annuity which pays 2 PLN at the beginning of each year for 4 years if the interest rate is 10% per annum continuously compounded. Rework example with deposits at the end of each year.
29. An investor borrowed 100 PLN. The loan was for four quarters at 20% annual interest. Create a loan amortization schedule for:
 - a) equal principal payments if interest payment is paid as a percent of the previous principal balance,
 - b) equal principal payments if interest payment is paid as a percent of the repaid loan,
 - c) equal total payments (compound interest rate),
 - d) equal total payments (continuously compounded interest rate).
30. An investor borrowed 1000 PLN. The loan was for 6 months at 24% annual interest (compound interest rate). Create a loan amortization schedule if
 - a) since the fourth month the annual interest is 18%,
 - b) the investor pays additional 100 PLN with the third payment,
 - c) the investor doesn't pay the fourth payment but he pays it plus interest with the fifth payment,
 - d) the first payment is postponed for two months,
 - e) the investor pays two payments, than he doesn't pay for 3 months. The investor begins to pay off the loan again in the sixth month paying three equal payments every two months. Since the third month the annual interest rate is 18%.
31. An investor borrowed 50 PLN. Find how many payments of 15 PLN should be made if the effective rate of interest is 10%. Solve the problem of non-integer number of payments.
32. An investor has to pay 12 monthly payments of 10 PLN (15% annual interest rate, compounding quarterly) and 5 semi-annual payments of 100 PLN (12% annual interest rate, compounding monthly). Change the debts into 10 quarterly payments of consolidated loan (18% annual interest rate, compounding annually).
33. Investor buys Treasury bills at the primary market with maturity 13 weeks and pays 97.9005 per a 100 PLN. Calculate the rate of return and the discount rate of the Treasury bills.

Sources:

1. Lecture notes <https://prawo.uni.wroc.pl/user/12141/students-resources>
2. Hastings K.J., *Introduction to Financial Mathematics*, CRC Press, 2015.