Mathematical Economics MME2/2 – 2017/2018 (lab)

1. A consumer has a utility function $u(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}} \right)^5$.

a) What are the Marshallian demand functions? (utility maximization)
b) What are the Hicksian demand functions? (expenditure minimization)

(1 point)

2. A firm has a production function given by $y = 5 x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$.
a) What are the factor demand functions?
b) What are the conditional factor demand functions?
c) What is the cost function?
d) What is the supply function?

(1 point)

3. The production function has the form $y(x_1, x_2) = A\left((1-a)x_1^{\rho} + a x_2^{\rho} \right)^{\frac{1}{\rho}}$, $\rho \neq 0$, $0 < a < 1$, $A > 0$. Compute:

a) the technical rate of substitution ($TRS = \dfrac{dx_2}{dx_1} = -\dfrac{\partial y/\partial x_1}{\partial y/\partial x_2}$)

b) the output elasticity of capital ($\varepsilon_1 = \dfrac{\partial y}{\partial x_1} \dfrac{x_1}{y} \equiv \dfrac{\partial \ln y}{\partial \ln x_1}$) and output elasticity of labour

($\varepsilon_2 = \dfrac{\partial y}{\partial x_2} \dfrac{x_2}{y}$),

c) the elasticity of substitution ($\sigma = \dfrac{d(x_2/x_1)}{dTRS} \dfrac{TRS}{(x_2/x_1)} \equiv \dfrac{d \ln(x_2/x_1)}{d \ln(TRS)}$),

d) the elasticity of scale ($\varepsilon_t = \lim\limits_{t \to 1} \dfrac{dy(tx_1, tx_2)}{dt} \dfrac{t}{y(tx_1, tx_2)}$),

e) $\lim\limits_{\rho \to 0} y$.

(1 point)

4. Suppose that we have two firms that face linear demand curve $p = 200 - \dfrac{1}{2}(y_1 + y_2)$ and

their cost functions are $c_1(y_1) = \dfrac{1}{2} y_1^2$, $c_2(y_2) = 10 y_2$, respectively.

a) Compute the Cournot equilibrium amount of output for each firm and firms' profits.
b) If firm 2 behaves as a follower and firm 1 behaves as a leader, compute the Stackelberg equilibrium amount of output for each firm and firms' profits.

(1 point)

5. The traders' utilities are given by $u^1(x_1, x_2) = x_1 x_2^2$ and $u^2(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Their initial endowments are the following $a^1 = (2,2)$ and $a^2 = (4,4)$. Traders come to a market and exchange commodities to maximize their utilities. Compute the price vector in equilibrium. Compare the utilities before and after the exchange.

(1 point)