## Mathematical Economics MME2/2 – 2017/2018 (final exam)

1. Find the demanded bundle for a consumer if the utility function and budget constraint are the following:

a) $u(x_1, x_2) = \left(\frac{1}{2}x_1 + 1\right)(x_2 + 2)$, $\quad 2x_1 + x_2 = 8$

b) $u(x_1, x_2) = \min\{3x_1 + x_2,\ x_1 + 3x_2\}$, $\quad x_1 + 2x_2 = 6$

c) $u(x_1, x_2) = 4x_1 + x_2$, $\quad 2x_1 + x_2 = 8$

(1 point)

2. A monopoly has a production function $y(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$. Solve the profit maximization problem and cost minimization problem in long run if $p(y) = 6y^{-\frac{1}{2}}$, $v_1 = 2$ and $v_2 = 3$.

(1 point)

3. For the technologies $y = \min\left\{\frac{1}{4}x_1, 2x_2\right\}$ and $y = \frac{x_1}{2} + x_2$ compute:

a) the total cost and the cost of production of 1 unit of output,

b) the marginal and average cost.

Assume that prices of inputs are (4, 6).

(1 point)

4. Check the returns to scale for the following technologies:

a) $f(x_1, x_2) = \sqrt{x_1 + 2x_2}$, $\quad$ b) $f(x_1, x_2) = x_1^{1/4} x_2^{3/4}$, $\quad$ c) $f(x_1, x_2) = \sqrt{x_1} + x_2^2$.

(1 point)

5. The production function has the form $y(x_1, x_2) = A\left((1-a)x_1^{\rho} + a x_2^{\rho}\right)^{\frac{1}{\rho}}$, $\rho \neq 0$, $0 < a < 1$, $A > 0$. Compute:

a) the technical rate of substitution

b) the output elasticity of capital (output elasticity of labour)

c) $\lim_{\rho \to 0} y$.

(1 point)