

# **Fundamentals of Financial Arithmetic**

## Lecture 5

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- **Long-term loans – repayment methods**
- Equal principal payments per time period
- Equal total payments per time period

- Loan amount – the size or value of the loan
- Interest rate – the annual stated rate of the loan
- Number of payments – the total numbers of payments to pay off the given loan amount
- Payment frequency – loans payments are due monthly (quarterly, annually).
- Compounding coincides with payments  
(Compounding doesn't coincide with payments)

- **Loan payment = principal payment + interest payment**
- The amortization schedule shows – for each payment – how much of the payment goes toward the loan principal, and how much is paid on interest.

## Example 1 – Loan Amortization Schedule

- An investor borrowed 100 PLN. The loan was for four quarters at 20% annual interest (compounding quarterly).

$$S = 100$$

$$N = 4$$

$$r = \frac{0.2}{4} = 0.05$$

**Loan amortization schedule – equal principal payments**  
 (interest payment as a percent of the previous principal balance)

$n$	$S_{n-1}$	$T_n$	$Z_n$	$A_n$	$S_n$
1	100	25	5	30	75
2	75	25	3.75	28.75	50
3	50	25	2.5	27.5	25
4	25	25	1.25	26.25	0
<b>Total</b>		<b>100</b>	<b>12.5</b>	<b>112.5</b>	

Previous principal balance    Principal payment    Interest payment    Total payment    Principal balance

## Loan amortization schedule – equal principal payments (interest payment as a percent of the repaid loan)

$n$	$S_{n-1}$	$T_n$	$Z_n$	$A_n$	$S_n$
1	100	25	1.25	26.25	75
2	75	25	2.5	27.5	50
3	50	25	3.75	28.75	25
4	25	25	5	30	0
<b>Total</b>		<b>100</b>	<b>12.5</b>	<b>112.5</b>	

Previous principal balance	Principal payment	Interest payment	Total payment	Principal balance
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**Loan amortization schedule – given principal payments**  
 (interest payment as a percent of the previous principal balance)

$n$	$S_{n-1}$	$T_n$	$Z_n$	$A_n$	$S_n$
1	100	10	5	15	90
2	90	20	4.5	24.5	70
3	70	20	3.5	23.5	50
4	50	50	2.5	52.5	0
<b>Total</b>		<b>100</b>	<b>15.5</b>	<b>115.5</b>	

Previous principal balance    Principal payment    Interest payment    Total payment    Principal balance



# Loan amortization schedule

$n$	$S_{n-1}$	$T_n$	$Z_n$	$A_n$	$S_n$
1	100	0	5	5	100
2	100	0	5	5	100
3	100	0	5	5	100
4	100	100	5	105	0
<b>Total</b>		<b>100</b>	<b>20</b>	<b>120</b>	

Previous  
principal  
balance

Principal  
payment

Interest  
payment

Total  
payment

Principal  
balance

## Equal total payments

$$S(1+r)^N = A_1(1+r)^{N-1} + A_2(1+r)^{N-2} + \dots + A_N$$

$$S = \frac{A_1}{1+r} + \frac{A_2}{(1+r)^2} + \dots + \frac{A_N}{(1+r)^N}$$

Periodic payment

$$S(1+r)^N = A \frac{(1+r)^N - 1}{r}$$

$$A = \frac{S \cdot r \cdot (1+r)^N}{(1+r)^N - 1}$$

## Equal total payments

$$Z_n = r \cdot S_{n-1} \quad T_n = S_{n-1} - S_n \quad A_n = T_n + Z_n$$

$$S_n = S(1+r)^n - \left( A_1(1+r)^{n-1} + A_2(1+r)^{n-2} + \dots + A_{n-1}(1+r) \right) - A_n$$

$$S_n = (1+r) \left( S(1+r)^{n-1} - \left( A_1(1+r)^{n-2} + A_2(1+r)^{n-3} + \dots + A_{n-1} \right) \right) - A_n$$

$$S_n = (1+r)S_{n-1} - A_n$$

**Loan amortization schedule – equal total payments**  
 (interest payment as a percent of the previous principal balance)

$n$	$S_{n-1}$	$T_n$	$Z_n$	$A_n$	$S_n$
1	100	23.2	5	28.2	76.8
2	76.8	24.36	3.84	28.2	52.44
3	52.44	25.58	2.62	28.2	26.86
4	26.86	26.86	1.34	28.2	0
<b>Total</b>		<b>100</b>	<b>12.8</b>	<b>112.8</b>	

Previous principal balance    Principal payment    Interest payment    Total payment    Principal balance

**Loan amortization schedule – given total payments**  
 (interest payment as a percent of the previous principal balance)

$n$	$S_{n-1}$	$T_n$	$Z_n$	$A_n$	$S_n$
1	100	15	5	20	85
2	85	25.75	4.25	30	59.25
3	59.25	37.04	2.96	40	22.21
4	22.21	22.21	1.11	23.32	0
Total		<b>100</b>	<b>13.32</b>	<b>113.32</b>	

Previous principal balance    Principal payment    Interest payment    Total payment    Principal balance

$$S(1+r)^4 = A_1(1+r)^3 + A_2(1+r)^2 + A_3(1+r) + \boxed{A_4}$$

# Equal total payments

(continuously compounded interest)

$$Se^{r \cdot N} = A_1 e^{r(N-1)} + A_2 e^{r(N-2)} + \dots + A_N$$

$$Se^{r \cdot N} = A \frac{e^{r \cdot N} - 1}{e^r - 1}$$

$$A = S \cdot e^{r \cdot N} \cdot \frac{e^r - 1}{e^{r \cdot N} - 1}$$

$$Z_n = S_{n-1} \cdot (e^r - 1) \quad T_n = S_{n-1} - S_n \quad A_n = T_n + Z_n$$

# Loan amortization schedule – equal total payments (continuously compounded interest)

$n$	$S_{n-1}$	$T_n$	$Z_n$	$A_n$	$S_n$
1	100	23.16	5.13	28.28	76.84
2	76.84	24.34	3.94	28.28	52.5
3	52.5	25.59	2.69	28.28	26.91
4	26.91	26.91	1.38	28.28	0
Total		<b>100</b>	<b>13.14</b>	<b>113.14</b>	

Previous  
principal  
balance

Principal  
payment

Interest  
payment

Total  
payment

Principal  
balance