Fundamentals of Financial Arithmetic
## Lecture 6

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Treasury bills

- Treasury bills are discounted short-term debt securities with maturities of up to one year.
- Treasury bills are sold at a discount off their nominal value.
- Treasury bills represent an important instrument of governmental fiscal policy and the central bank's monetary policy.
- The nominal value is payable to the final holder upon redemption on maturity.
- Nominal/face value – 10 000 PLN in Poland.
- Maturity – the date the bill is redeemed and the investor is paid the face value amount.
- Regular Treasury bill series are issued weekly (13, 26 or 52 weeks in Poland).

Bill valuation methods

$$t$$

$$P_1 \qquad\qquad\qquad P_2$$
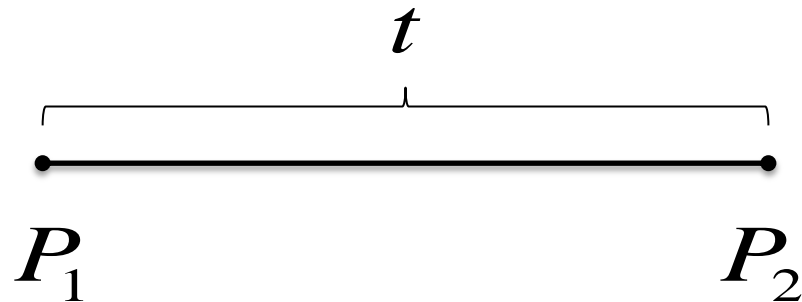
- P_1 – purchase price (at which investor can buy)
- P_2 – nominal/face value (principal)
- t – number o days from purchase to maturity

Bill valuation methods

- The method applied to determine the value of bills depends on whether the bill price is based on the rate of return (r) or the rate of discount (d).

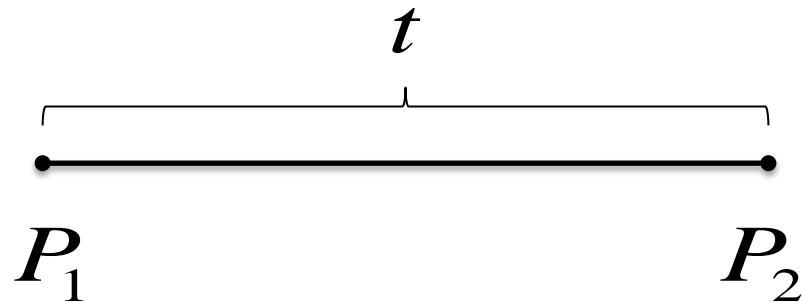- Bond prices are quoted relative to a 100 PLN face/nominal value.

Treasury bills – the rate of return

$$t$$
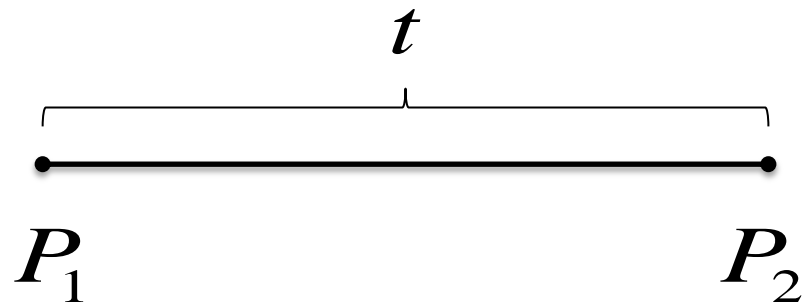
P_1 $\qquad\qquad\qquad\qquad$ P_2

$$\frac{P_2 - P_1}{P_1} \quad \rightarrow \quad t$$

$$r \qquad \rightarrow \quad 360$$
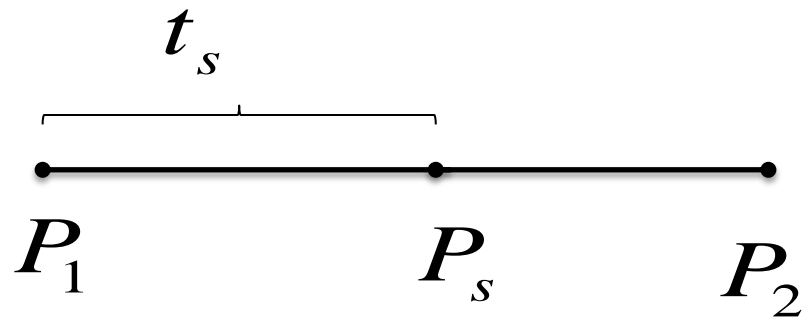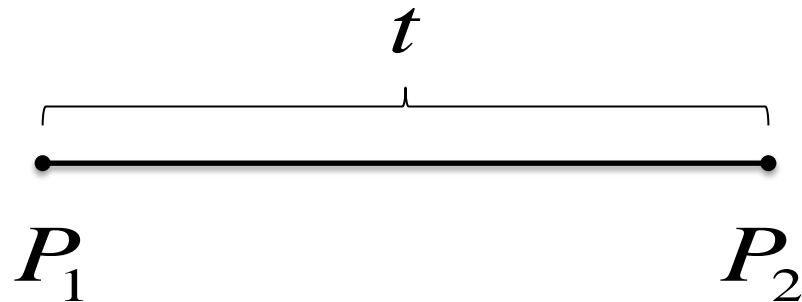
Treasury bills – the rate of return

$$r = \frac{P_2 - P_1}{P_1} \cdot \frac{360}{t}$$

Treasury bills – the rate of return for the holding period

$$r_s = \frac{P_s - P_1}{P_1} \cdot \frac{360}{t_s}$$

Treasury bills – the discount rate

$$t$$

$$P_1 \qquad\qquad P_2$$

$$d = \frac{P_2 - P_1}{P_2} \cdot \frac{360}{t}$$

Treasury bills – price of the Treasury bills

- The price per 100 PLN principal (bills quoted on the basis of the rate of return).

$$P = \frac{360}{r \cdot t + 360} \cdot 100$$

- The price per 100 PLN principal (bills quoted on the basis of the discount rate)

$$P = \left(1 - \frac{d \cdot t}{360}\right) \cdot 100$$

Treasury bills

$$\frac{360}{r \cdot t + 360} \cdot 100 = \left(1 - \frac{d \cdot t}{360}\right) \cdot 100$$

$$r = \frac{d}{1 - d \cdot \dfrac{t}{360}}$$

$$d = \frac{r}{1 + r \cdot \dfrac{t}{360}}$$

The rate of return for the known discount rate

The discount rate for the known rate of return

Example 1 – Treasury bills

Investor buys Treasury bills at the primary market with maturity 26 weeks. The nominal value of bills is 1.5 million PLN. The investors pays 97.9005 per a 100 PLN.

$$9790.05 \cdot 150 = 1468508$$

- The rate of return

$$r = \frac{100 - 97.9005}{97.9005} \cdot \frac{360}{182} = 0.04242$$

- The discount rate

$$d = \frac{100 - 97.9005}{100} \cdot \frac{360}{182} = 0.04153$$

Example 2 – Treasury bills

- Assuming that the Treasury bills have been issued at a rate of return of 9% per 60 days, calculate the appropriate discount rate.

$$d = \frac{r}{1 + r \cdot \dfrac{t}{360}} = \frac{0.09}{1 + 0.09 \cdot \dfrac{60}{360}} = 0.08867$$

A certificate of deposit – CD

- A certificate of deposit is a savings certificate with a fixed maturity date, specified fixed interest rate issued by commercial banks.

- A CD restricts access to the funds until the maturity date of the investment.
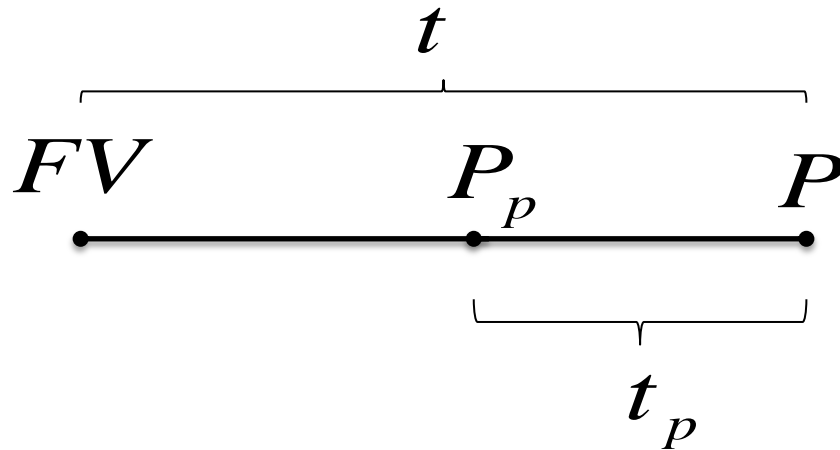
A certificate of deposit

$$FV \underset{\text{Face value}}{\bullet} \overbrace{\rule{4cm}{0pt}}^{t} \underset{\text{Price at maturity}}{\bullet} P$$
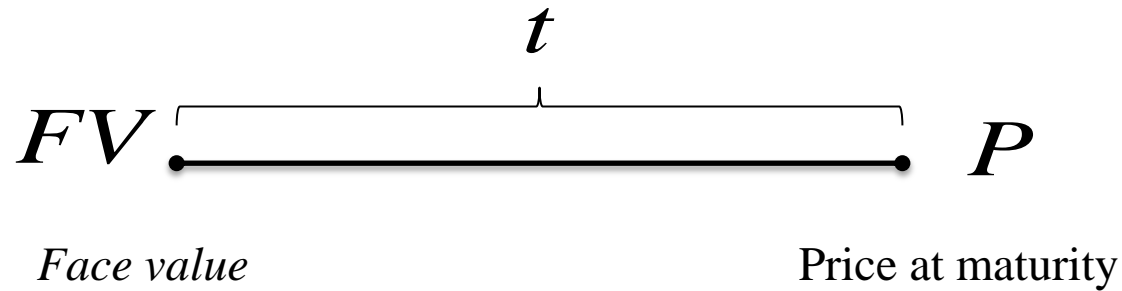
$$P = FV \cdot \left(1 + r_k \cdot \frac{t}{360} \right)$$

r_k – interest rate

A certificate of deposit

$$t$$

$$FV \qquad P_p \qquad P$$

$$t_p$$

Number of days
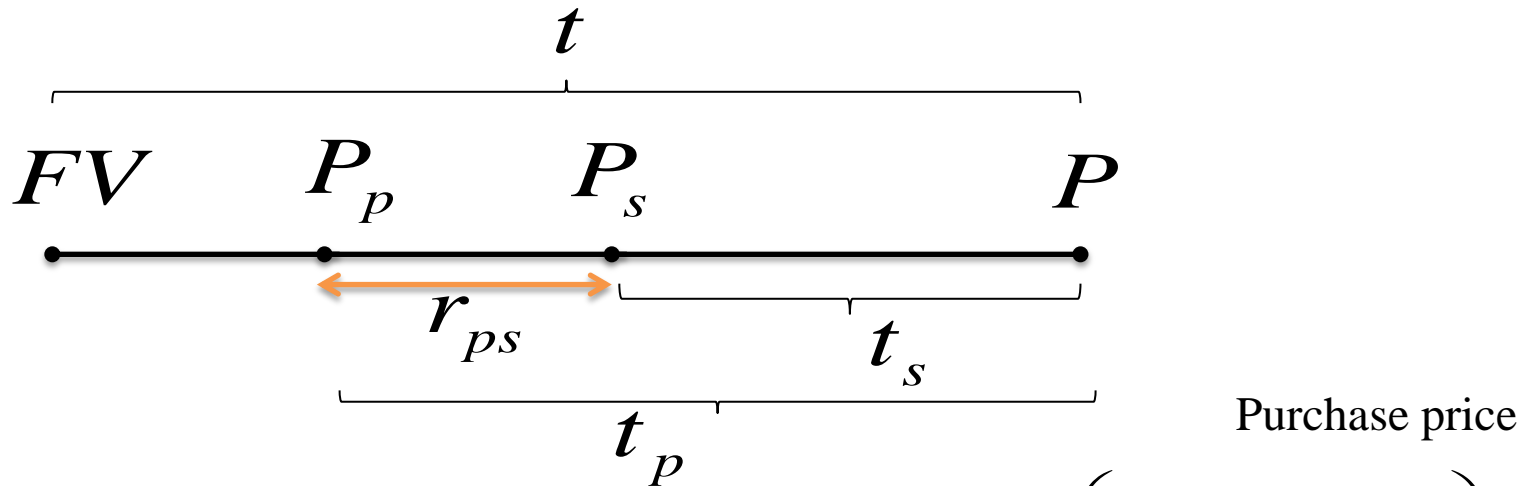from purchase to maturity

$$FV \cdot \left(1 + r_k \cdot \frac{t}{360} \right) = P_p \cdot \left(1 + r_p \cdot \frac{t_p}{360} \right)$$

$$P_p = \frac{FV \cdot \left(1 + r_k \cdot \dfrac{t}{360} \right)}{\left(1 + r_p \cdot \dfrac{t_p}{360} \right)}$$

$$P_p = \frac{100 \cdot \left(1 + r_k \cdot \dfrac{t}{360} \right)}{\left(1 + r_p \cdot \dfrac{t_p}{360} \right)}$$

Purchase price

CD – the rate of return for the holding period

$$r_{ps} = \frac{P_s - P_p}{P_p} \cdot \frac{360}{t_p - t_s}$$

Purchase price

$$P_p = \frac{100 \cdot \left(1 + r_k \cdot \dfrac{t}{360} \right)}{\left(1 + r_p \cdot \dfrac{t_p}{360} \right)}$$
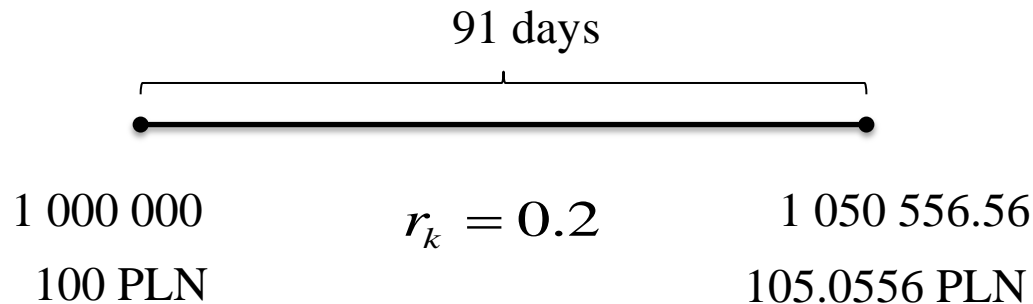
$$r_{ps} = \left(\frac{1 + r_p \cdot \dfrac{t_p}{360}}{1 + r_s \cdot \dfrac{t_s}{360}} - 1 \right) \cdot \frac{360}{t_p - t_s}$$

$$P_s = \frac{100 \cdot \left(1 + r_k \cdot \dfrac{t}{360} \right)}{\left(1 + r_s \cdot \dfrac{t_s}{360} \right)}$$

Sell price

Example 3 – CD

- Investor buys CD at the primary market with maturity 13 weeks. The nominal value of CD is 1 million PLN. The rate of return is 20%.

- Calculate the price at maturity

91 days

1 000 000 $r_k = 0.2$ 1 050 556.56

100 PLN   105.0556 PLN

$$P = 1000000 \cdot \left(1 + 0.2 \cdot \frac{91}{360}\right) = 1050556.556$$

Example 3 – CD

- After 31 days the investor sells CD at a 19.75% rate of return.

$$31 \text{ days} \qquad 60 \text{ days}$$

$$1\ 000\ 000 \qquad 1\ 017\ 076.8 \qquad 1\ 050\ 556.56$$
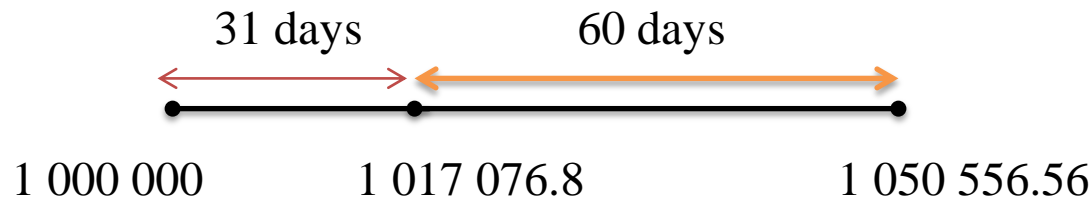
$$P_s = \frac{1000000 \cdot \left(1 + 0.2 \cdot \dfrac{91}{360}\right)}{\left(1 + 0.1975 \cdot \dfrac{60}{360}\right)} = 1017076.8$$

101.7077 – dirty price

101.7077-1.7222 = 99.9855 – clean price

Interest for 100 PLN

$$100 \cdot \frac{0.2 \cdot 31}{360} = 1.722$$

1 017 076.8-17 222.2= 999 854.6

-145,4 PLN

Example 3 – CD

31 days   60 days

1 000 000   20%   ? %   1 050 556.56
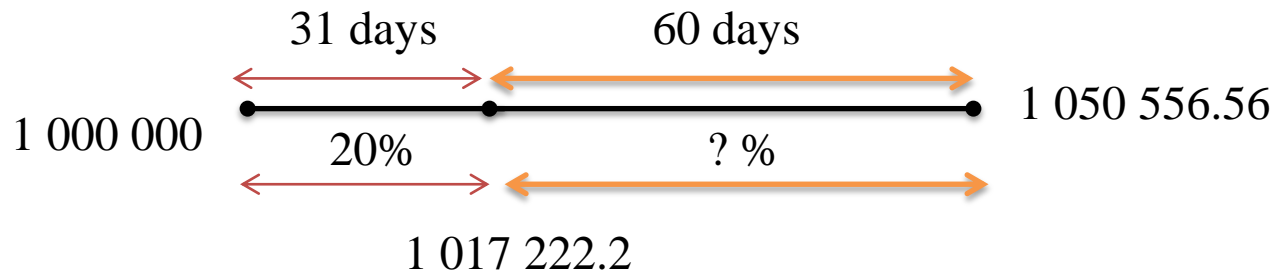
1 017 222.2

$$P_s = 1000000 \cdot \left(1 + 0.2 \cdot \frac{31}{360}\right) = 1017222.2$$

$$r_s = \frac{1050556.56 - 1017222.2}{1017222.2} \cdot \frac{360}{60} = \boxed{0.1966}$$