

Fundamentals of Financial Arithmetic

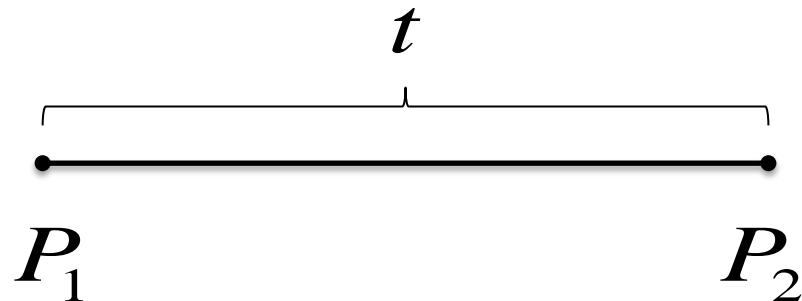
Lecture 7

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Treasury bills

- Treasury bills are discounted short-term debt securities with maturities of up to one year.
- Treasury bills are sold at a discount off their nominal value.
- Treasury bills represent an important instrument of governmental fiscal policy and the central bank's monetary policy.
- The nominal value is payable to the final holder upon redemption on maturity.
- Nominal/face value – 10 000 PLN in Poland.
- Maturity – the date the bill is redeemed and the investor is paid the face value amount.
- Regular Treasury bill series are issued weekly (13, 26 or 52 weeks in Poland).

Bill valuation methods

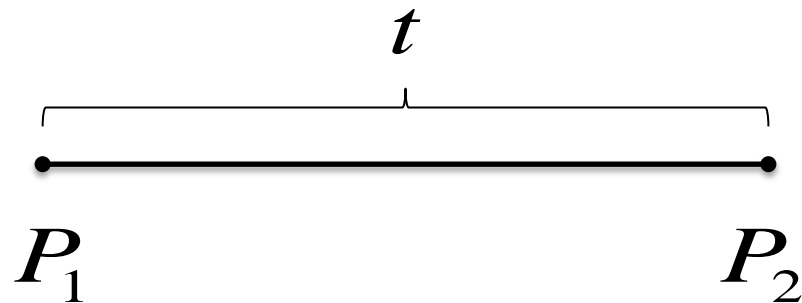


- P_1 – purchase price (at which investor can buy)
- P_2 – nominal/face value (principal)
- t – number of days from purchase to maturity

Bill valuation methods

- The method applied to determine the value of bills depends on whether the bill price is based on the rate of return (r) or the rate of discount (d).
- Bill prices are quoted relative to a 100 PLN face/nominal value.

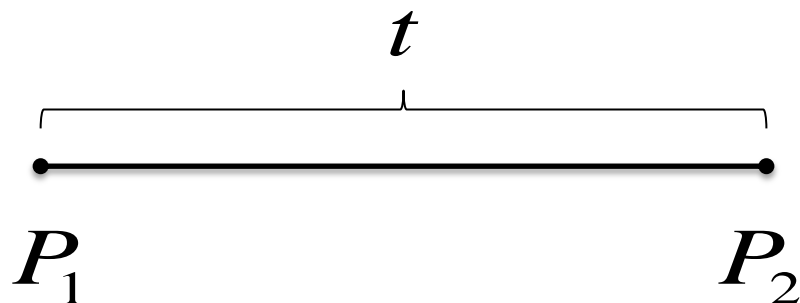
Treasury bills – the rate of return



$$\frac{P_2 - P_1}{P_1} \rightarrow t$$

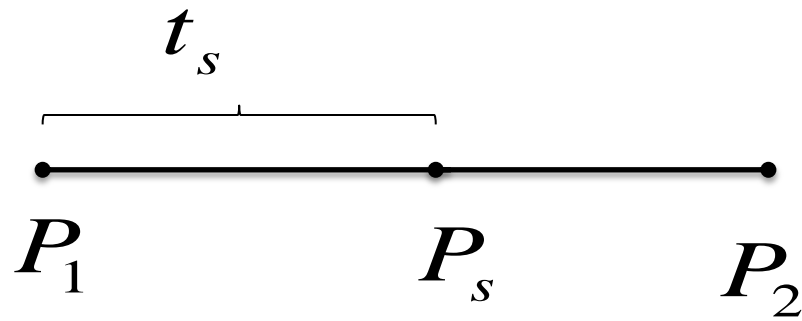
$$r \rightarrow 360$$

Treasury bills – the rate of return



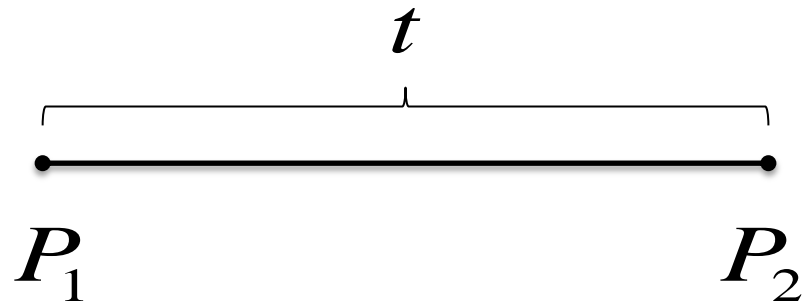
$$r = \frac{P_2 - P_1}{P_1} \cdot \frac{360}{t}$$

Treasury bills – the rate of return for the holding period



$$r_s = \frac{P_s - P_1}{P_1} \cdot \frac{360}{t_s}$$

Treasury bills – the discount rate



$$d = \frac{P_2 - P_1}{P_2} \cdot \frac{360}{t}$$

Treasury bills – price of the Treasury bills

- The price per 100 PLN principal (bills quoted on the basis of the rate of return).

$$P = \frac{360}{r \cdot t + 360} \cdot 100$$

- The price per 100 PLN principal (bills quoted on the basis of the discount rate)

$$P = \left(1 - \frac{d \cdot t}{360} \right) \cdot 100$$

Treasury bills

$$\frac{360}{r \cdot t + 360} \cdot 100 = \left(1 - \frac{d \cdot t}{360} \right) \cdot 100$$

$$r = \frac{d}{1 - d \cdot \frac{t}{360}}$$

The rate of return for the known discount rate

$$d = \frac{r}{1 + r \cdot \frac{t}{360}}$$

The discount rate for the known rate of return

Example 1 – Treasury bills

Investor buys Treasury bills at the primary market with maturity 26 weeks. The nominal value of bills is 1.5 million PLN. The investors pays 97.9005 per a 100 PLN.

$$9790.05 \cdot 150 = 1468508$$

- The rate of return

$$r = \frac{100 - 97.9005}{97.9005} \cdot \frac{360}{182} = 0.04242$$

- The discount rate

$$d = \frac{100 - 97.9005}{100} \cdot \frac{360}{182} = 0.04153$$

Example 2 – Treasury bills

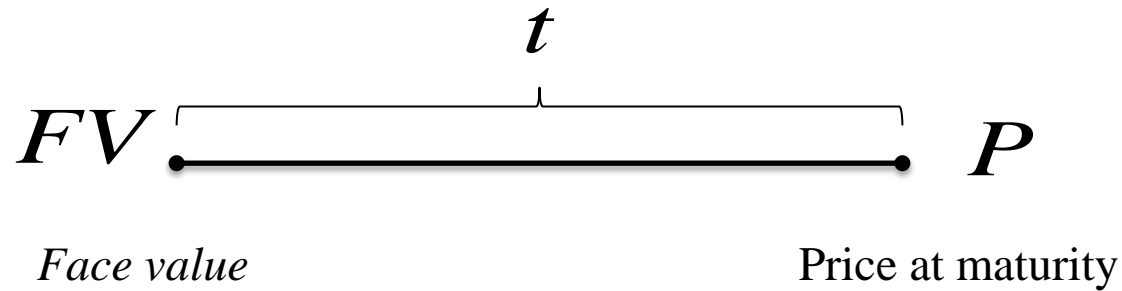
- Assuming that the Treasury bills have been issued at a rate of return of 9% per 60 days, calculate the appropriate discount rate.

$$d = \frac{r}{1 + r \cdot \frac{t}{360}} = \frac{0.09}{1 + 0.09 \cdot \frac{60}{360}} = 0.08867$$

A certificate of deposit – CD

- A certificate of deposit is a savings certificate with a fixed maturity date, specified fixed interest rate issued by commercial banks.
- A CD restricts access to the funds until the maturity date of the investment.

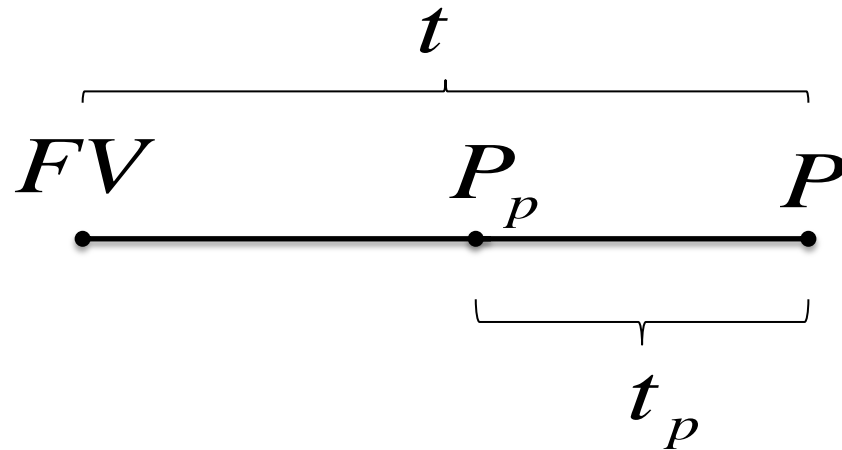
A certificate of deposit



$$P = FV \cdot \left(1 + r_k \cdot \frac{t}{360} \right)$$

r_k – interest rate

A certificate of deposit



Number of days
from purchase to maturity

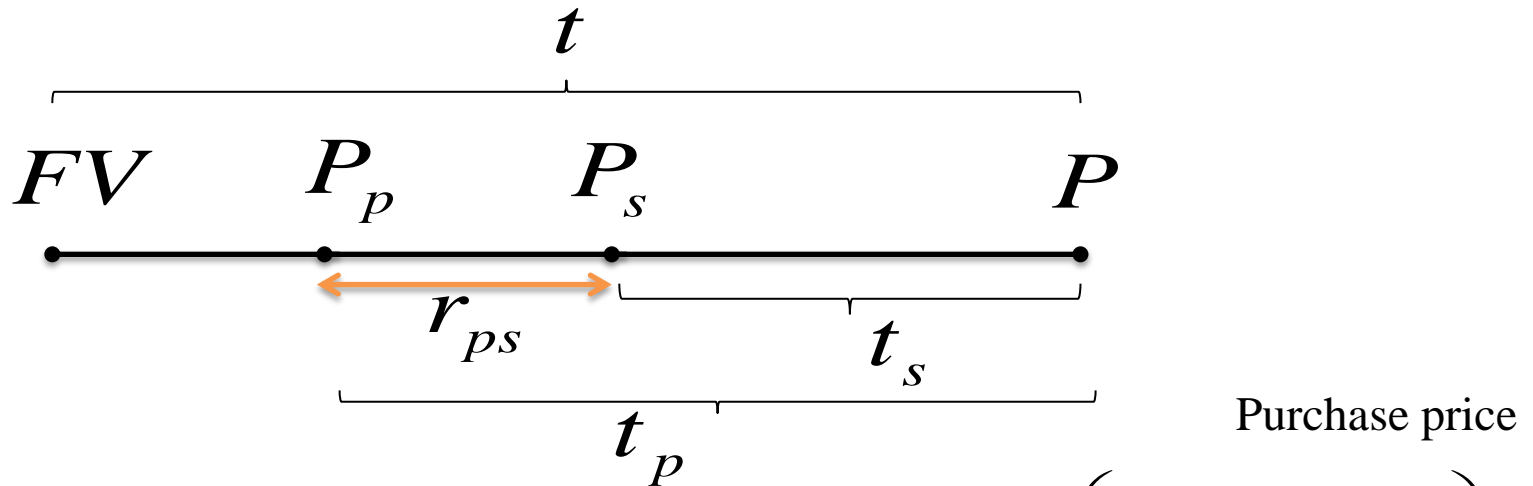
$$FV \cdot \left(1 + r_k \cdot \frac{t}{360}\right) = P_p \cdot \left(1 + r_p \cdot \frac{t_p}{360}\right)$$

$$P_p = \frac{FV \cdot \left(1 + r_k \cdot \frac{t}{360}\right)}{\left(1 + r_p \cdot \frac{t_p}{360}\right)}$$

Purchase price

$$P_p = \frac{100 \cdot \left(1 + r_k \cdot \frac{t}{360}\right)}{\left(1 + r_p \cdot \frac{t_p}{360}\right)}$$

CD – the rate of return for the holding period



$$r_{ps} = \frac{P_s - P_p}{P_p} \cdot \frac{360}{t_p - t_s}$$

$$P_p = \frac{100 \cdot \left(1 + r_k \cdot \frac{t}{360}\right)}{\left(1 + r_p \cdot \frac{t_p}{360}\right)}$$

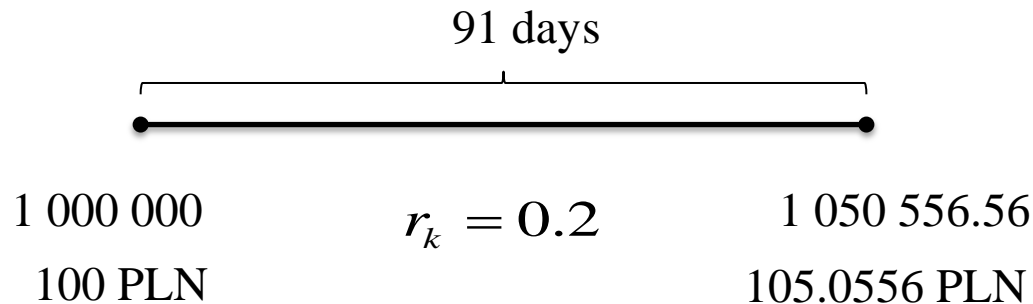
$$r_{ps} = \left(\frac{1 + r_p \cdot \frac{t_p}{360}}{1 + r_s \cdot \frac{t_s}{360}} - 1 \right) \cdot \frac{360}{t_p - t_s}$$

$$P_s = \frac{100 \cdot \left(1 + r_k \cdot \frac{t}{360}\right)}{\left(1 + r_s \cdot \frac{t_s}{360}\right)}$$

Sell price

Example 3 – CD

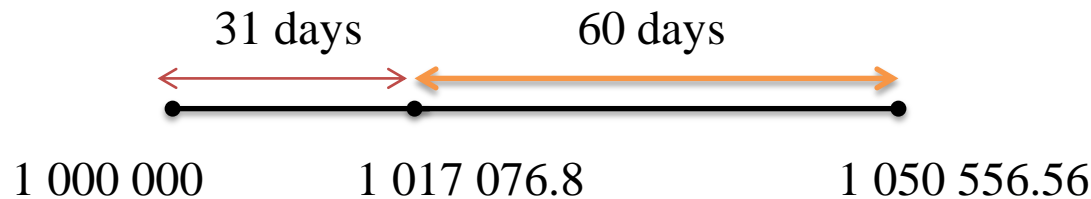
- Investor buys CD at the primary market with maturity 13 weeks. The nominal value of CD is 1 million PLN. The rate of return is 20%.
- Calculate the price at maturity



$$P = 10000000 \cdot \left(1 + 0.2 \cdot \frac{91}{360} \right) = 1050556.556$$

Example 3 – CD

- After 31 days the investor sells CD at a 19.75% rate of return.



$$P_s = \frac{10000000 \cdot \left(1 + 0.2 \cdot \frac{91}{360}\right)}{\left(1 + 0.1975 \cdot \frac{60}{360}\right)} = 1017076.8$$

Interest for 100 PLN

$$100 \cdot \frac{0.2 \cdot 31}{360} = 1.722$$

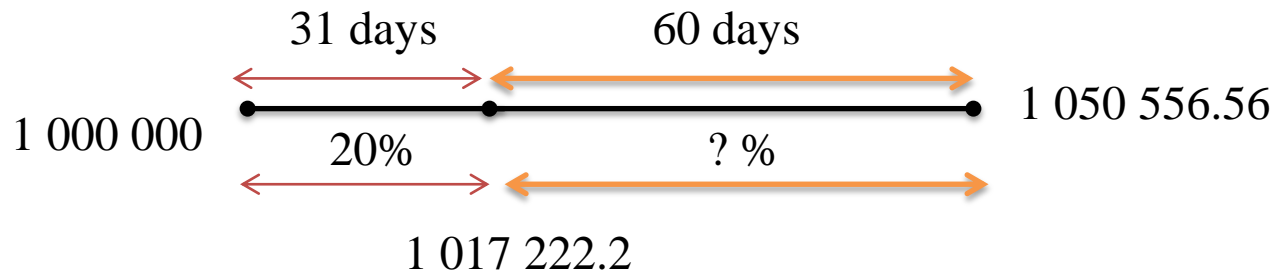
101.7077 – dirty price

$$1\ 017\ 076.8 - 17\ 222.2 = 999\ 854.6$$

$$101.7077 - 1.7222 = 99.9855 \text{ – clean price}$$

-145,4 PLN

Example 3 – CD



$$P_s = 1000000 \cdot \left(1 + 0.2 \cdot \frac{31}{360} \right) = 1017222.2$$

$$r_s = \frac{1050556.56 - 1017222.2}{1017222.2} \cdot \frac{360}{60} = \boxed{0.1966}$$