Fundamentals of Financial Arithmetic

Lecture 8

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Fundamentals of bond valuation

- Bond – a loan between a borrower (issuer) and a lender (investor, creditor)
- The issuer promises to make regular interest payments to the investor at a specified rate (the **coupon rate**) on the amount it has borrowed (the **face/par amount**) until a specified date (the **maturity date**).
- Once the bond matures, the interest payments stop and the issuer is required to repay the face amount of the principal to the investor.

Fundamentals of bond valuation

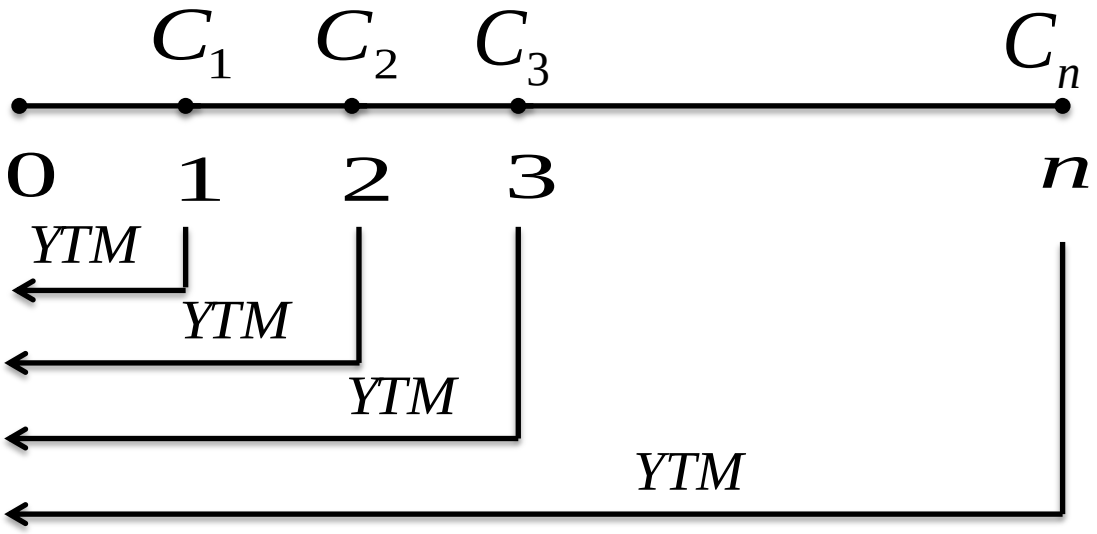
- Bonds can be priced at **a premium**, **discount**, or **at par**.
- If the bond's price is higher than its par value, it will sell at a premium because its interest rate is higher than current prevailing rates.
- If the bond's price is lower than its par value, the bond will sell at a discount because its interest rate is lower than current prevailing interest rates.

Fundamentals of bond valuation

- Bond valuation is the determination of the fair price of a bond.
- The price of bond is the sum of the present values of all expected coupon payments plus the present value of the par value at maturity.
- Yield to maturity – is the internal rate of return earned by investor who buys the bond today at the market price, assuming that the bond will be held until maturity.

Bond pricing – coupon bonds

- C_i – income from the ownership bonds at time i, n – number of payments, YTM – yield to maturity, P – bond price



$$P = \frac{C_1}{1+YTM} + \frac{C_2}{(1+YTM)^2} + \quad \cdots \quad + \frac{C_n}{(1+YTM)^n} = \sum_{i=1}^{n} \frac{C_i}{(1+YTM)^i}$$

Bond pricing – coupon bonds

- **Constant coupon rate**, C – coupon payment, M – value at maturity or par value, n – number of payments, YTM – yield to maturity, P – bond price

$$P = \frac{C}{1+YTM} + \frac{C}{(1+YTM)^2} + \cdots + \frac{C+M}{(1+YTM)^n}$$

$$P = \frac{C}{1+YTM}\left(1 + \frac{1}{1+YTM} + \cdots + \frac{1}{(1+YTM)^{n-1}}\right) + \frac{M}{(1+YTM)^n}$$

$$P = C \cdot \frac{1-(1+YTM)^{-n}}{YTM} + \frac{M}{(1+YTM)^n}$$

Example 1

Suppose a 4-year bond with the value at maturity of 100 PLN and a coupon rate of 10%.

Time to maturity	**Price of bond**			**Premium**	**Discount**	**Percent of premium decline**	**Percent of discount decline**
	YTM= 9%	YTM=10%	YTM=11%				
4	103.24	100	96.90	3.24	3.10	–	–
3	102.53	100	97.56	2.53	2.44	21.87%	21.23%
2	101.76	100	98.29	1.76	1.71	30.51%	29.92%
1	100.92	100	99.10	0.92	0.9	47.85%	47.39%

$$\frac{3.24-2.53}{3.24}=0.2187$$

Example 2

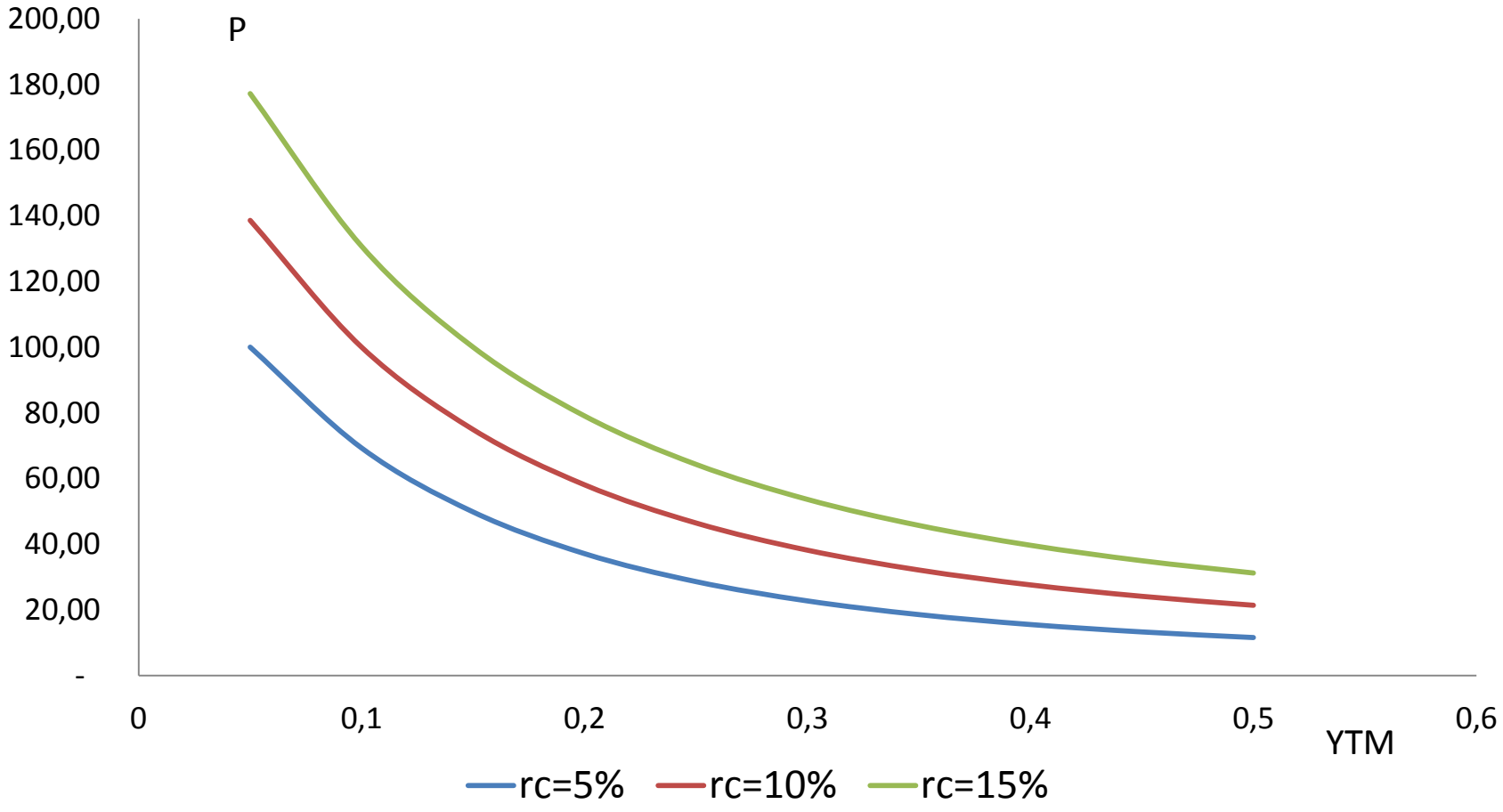
- Suppose a 3-year bond with the value at maturity of 100 PLN.

Coupon rate	**Price of bond**		**Percent of decrease**
	YTM = 8%	YTM = 12%	
10%	105.15	95.20	9.47%
15%	118.04	107.21	9.18%

$$\frac{105.15 - 95.2}{105.15} = 0.0947$$

Example 3

n=10 M=100



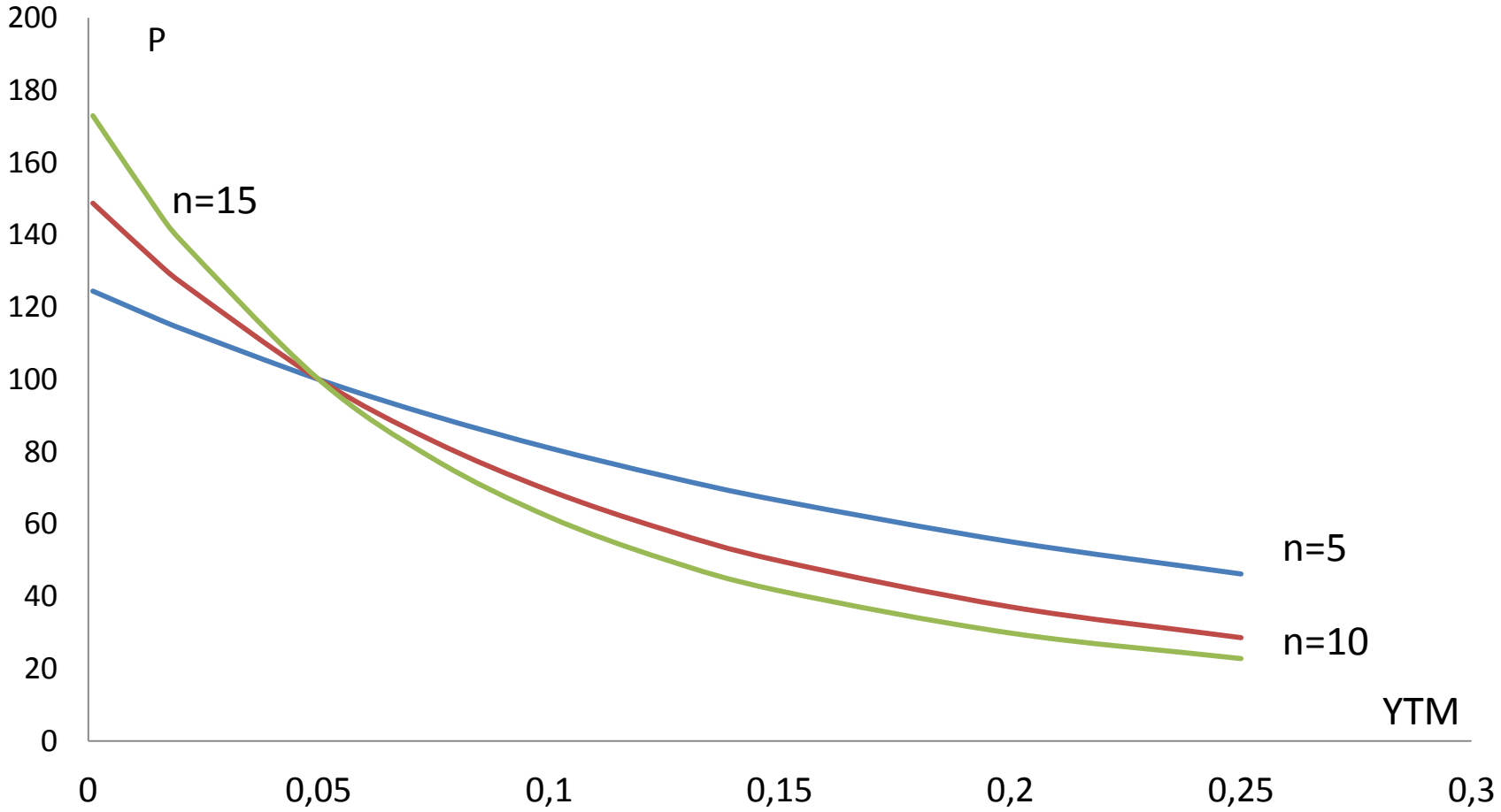
Example 4

- Suppose a bond with the value at maturity of 100 PLN and a coupon rate of 10%.

Time to maturity (in years)	**Price of bond**		**Percent of decrease**
	YTM = 8%	YTM = 12%	
3	105.15	95.20	9.47%
5	107.99	92.79	14.07%

Example 5

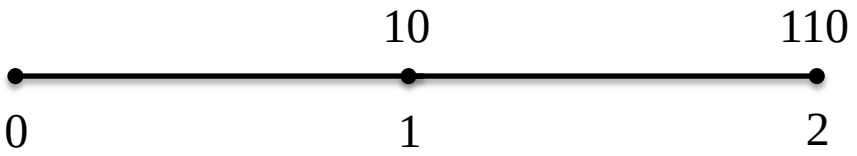
M=100, rc=5%



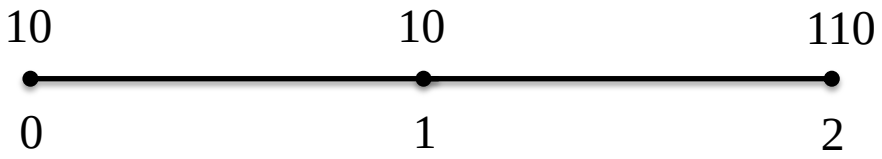
Example 6

- Calculate the price of a bond with a par value of 100 PLN to be paid in two years (after and before the coupon payment), a coupon rate of 10%, and a required yield of 9%.

$$P = \frac{10}{1.09} + \frac{110}{(1.09)^2} = 101.76$$

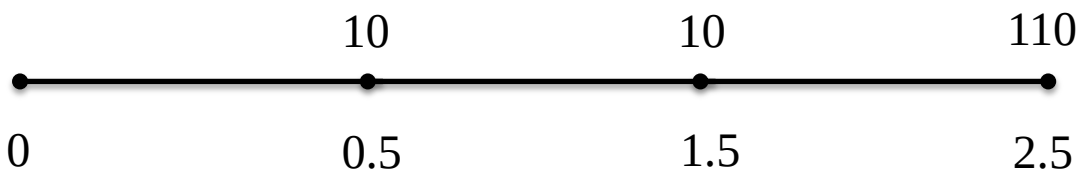


$$P = 10 + \frac{10}{1.09} + \frac{110}{(1.09)^2} = 111.76$$



Example 7

- Calculate the price of a bond with a par value of 100 PLN to be paid in two years and six months, a coupon rate of 10%, and a required yield of 8%. An annual coupon payment.



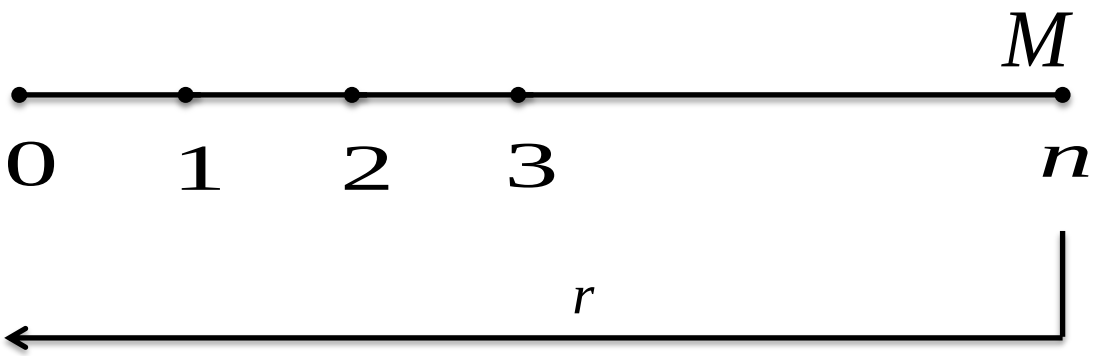
$$P = \frac{10}{(1.08)^{0.5}} + \frac{10}{(1.08)^{1.5}} + \frac{110}{(1.08)^{2.5}} = 109.28$$

Zero-coupon bonds

- Zero-coupon or accrual bonds do not pay a coupon. Instead, these types of bonds are issued at a deep discount and pay the full face value at maturity.

Fundamentals of bond valuation – bond price

- **Zero-coupon bond**, M – value at maturity, n – number of periods, r – interest rate, P – bond price



$$P = \frac{M}{(1+r)^n}$$

Example 8

- Calculate the price of a zero-coupon bond that is maturing in one and a half years, has a par value of 100 PLN and a required yield of 5%.

$$P = \frac{100}{(1+0.05)^{1.5}} = 92.94$$