Financial Mathematics
Lecture 11-12

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Derivatives

- Derivatives – financial contracts which value depends on the value of the underlying asset.

- The underlying asset can be **physical commodities** such as oil, metals (gold, platinum, copper, silver, uranium, aluminum), corn, live cattle, and so on or **financial instruments** as stocks, bonds, currencies, stock indexes, interest rate.

Long in position – short in **position**

- The buyer of the contract is **long** in position and the seller of the contract is **short** in position.

- The long position will take the delivery of the asset and pay the seller of the asset the contract value while the seller is obligated to deliver the assets versus the cash value of the contract at expiration.

The way the derivatives are traded in the market

- **Over-the-counter** (OTC) derivatives: derivatives that are traded (and privately negotiated) directly between two parties, without going through an exchange or other intermediary.

- **Exchange-trade derivatives**: standardized derivatives that are traded on stock exchanges or specialized derivatives exchanges.

Derivatives

Four types of derivatives stand out:

- Forward contracts,
- Futures contracts,
- Options,
- Swaps.

- A *forward contract* is an agreement between two parties in which one of the parties assumes a long position (the other party assumes a short position) and obliges to purchase (sell) the underlying asset at a specified future date, (*expiration date* or *maturity*) at a specified price (*delivery price*).

- A *futures contract* is standardized contract between two parties to buy or sell a specified asset of standardized quantity and quality for the price agreed upon today with delivery and payment occurring at a specified future date.

- Futures and forward contracts are obligations on both the buyers and the sellers.
- They can be used for speculation, hedging, or to arbitrage between the spot and the deferred-delivery markets.

- An *option* is a contract which gives the buyer the right but not the obligation, to buy or sell an underlying asset at a **strike price** on a specified date.

- Options are binding only on the sellers.

- The buyers have the right, but not obligation, to take a position in the underlying asset.
- Options can be used to hedge downside risk, speculation, or arbitrage markets.

Call option – put option

- **Call** options provide the holder the right to purchase an underlying asset at a specified price on a specified date.

- **Put** options give the holder the right to sell an underlying asset at a specified price on a specified date.

European option – American option

- Owners of American-style option may exercise at any time before the option expires, while owners of European-style option may exercise only at expiration.

Swaps

- Swaps are agreements between two counterparties to exchange cash flows in the future according to predetermined formula.
- There are two basic types of swaps: interest rate and currency.
- An interest rate swap occurs when two parties exchange interest payments periodically.
- Currency swaps are agreements to deliver one currency against another.

Valuation of forward/futures

- The underlying asset pays no income
- The underlying asset pays predictable income
- The underlying asset pays continuous dividend yields

- The fair (or theoretical) price of a forward contract
- The value of a forward contract

- Simple interest

$$K_n = K_0 \cdot (1 + n \cdot r)$$

- Compound interest

$$K_n = K_0 \cdot (1 + r)^n$$

- Continuously compounded interest

$$K_n = K_0 \cdot e^{n \cdot r}$$

The fair price of a forward contract

➢ For a forward contract on an underlying asset providing no income

$$\boxed{F = S_0 \cdot e^{r \cdot T}}$$

- F denotes the forward/futures price
- T denotes the maturity date of the contract
- r denotes the riskless interest rate
- S_0 denotes the spot price of the underlying asset

The fair price of a forward contract

- For a forward contract on an underlying asset providing a predictable income <u>with a present value</u> of D

$$\boxed{F = \left(S_0 - D\right) \cdot e^{r \cdot T}}$$

$$D = \sum_{i=1}^{n} D_i e^{-r \cdot t_i}$$

- D_i denotes the dividend paid out at time t_i

The fair price of a forward contract

- The underlying asset pays continuous dividend yields

$$\boxed{F = S_0 \cdot e^{(r-d)\cdot T}}$$

- A continuous dividend yield means dividends are paid out continuously at an annual rate of *d*.

Compound interest	**Simple interest**
The underlying asset pays no income	
$F = S_0 \cdot (1 + r)^T$	$F = S_0 \cdot (1 + T \cdot r)$
The underlying asset pays predictable income	
$F = (S_0 - PV) \cdot (1 + r)^T$	$F = (S_0 - PV) \cdot (1 + T \cdot r)$
The underlying asset pays continuous dividend yields	
$F = \frac{S_0 \cdot (1 + r)^T}{(1 + d)^T}$	$F = \frac{S_0 \cdot (1 + T \cdot r)}{1 + T \cdot d}$

Cash-and-carry arbitrage

- A cash-and-carry arbitrage occurs when the investor borrows money, buys the goods today for cash and carries the goods to the expiration of the forward contract.
- Then, delivers the commodity against a forward contract and pays off the loan.
- Any profit from this strategy would be an arbitrage profit.

The underlying asset pays no income

Time	Strategies	Cash flow
The seller of the forward contract ($F > S_0e^{rT}$)		
$t = 0$	The investor sells the contract (enters into short forward contract).	–
	The investor goes to the bank and borrows the amount S_0 for time T, at the continuously compounded risk-free rate r	$+S_0$
	The investor buys the underlying asset.	$-S_0$
Total cash flow		0
$t = T$	The investor settles the short forward contract by selling the asset for F.	$+F$
	The investor repays the loan to the bank.	$-S_0e^{rT}$
Total cash flow		$F - S_0e^{rT} > 0$

Reverse cash-and-carry arbitrage

- A reverse cash-and-carry arbitrage occurs when the investor <u>sells short</u> an asset.
- The investor purchases a forward contract, which will be used to honor the short sale commitment.
- Then the investor lends the proceeds at an established rate of interest.
- In the future, the investor accepts delivery against the forward contract and uses the commodity received to cover the short position.
- Any profit from this strategy would be an arbitrage profit.

The underlying asset pays no income

Time	Strategies	Cash flow
The buyer of the forward contract ($F < S_0 e^{rT}$)		
$t = 0$	The investor purchases the contract (enters into long forward contract).	–
	The investor sells short the underlying asset.	$+S_0$
	The investor invests the proceeds for T, at the continuously compounded rate r	$-S_0$
Total cash flow		0
$t = T$	The liquidation of the deposit.	$S_0 e^{rT}$
	The investor settles the long forward contract by purchasing the asset for F.	$-F$
	The investor closes out the short position.	–
Total cash flow		$S_0 e^{rT} - F > 0$

The underlying asset pays predictable income

Time	Strategies	Cash flow
The seller of the forward contract $(F > (S_0 - D)e^{rT})$		
$t = 0$	The investor sells the contract (enters into short forward contract).	$-$
	The investor goes to the bank and borrows the amount S_0 for time T, at the continuously compounded risk-free rate r	$+S_0$
	The investor buys the underlying asset.	$-S_0$
Total cash flow		0
$t = T$	The investor settles the short forward contract by selling the asset for F.	$+F$
	The investor repays the loan to the bank.	$-S_0e^{rT}$
	Return on invested dividends.	De^{rT}
Total cash flow		$F - S_0e^{rT} + De^{rT} > 0$

The underlying asset pays predictable income

Time	Strategies	Cash flow
The buyer of the forward contract ($F < (S_0 - D)e^{rT}$)		
$t = 0$	The investor purchases the contract (enters into long forward contract).	–
	The investor sells short the underlying asset.	$+S_0$
	The investor invests the proceeds for T, at the continuously compounded rate r	$-S_0$
Total cash flow		0
$t = T$	The liquidation of the deposit.	$S_0 e^{rT}$
	The investor settles the long forward contract by purchasing the asset for F.	$-F$
	The investor closes out the short position and returns dividends.	$-De^{rT}$
Total cash flow		$S_0 e^{rT} - F - De^{rT} > 0$

The value of a forward contract

- The underlying asset pays no income

$$f_t = S_t - F \cdot e^{-(T-t)\cdot r}$$

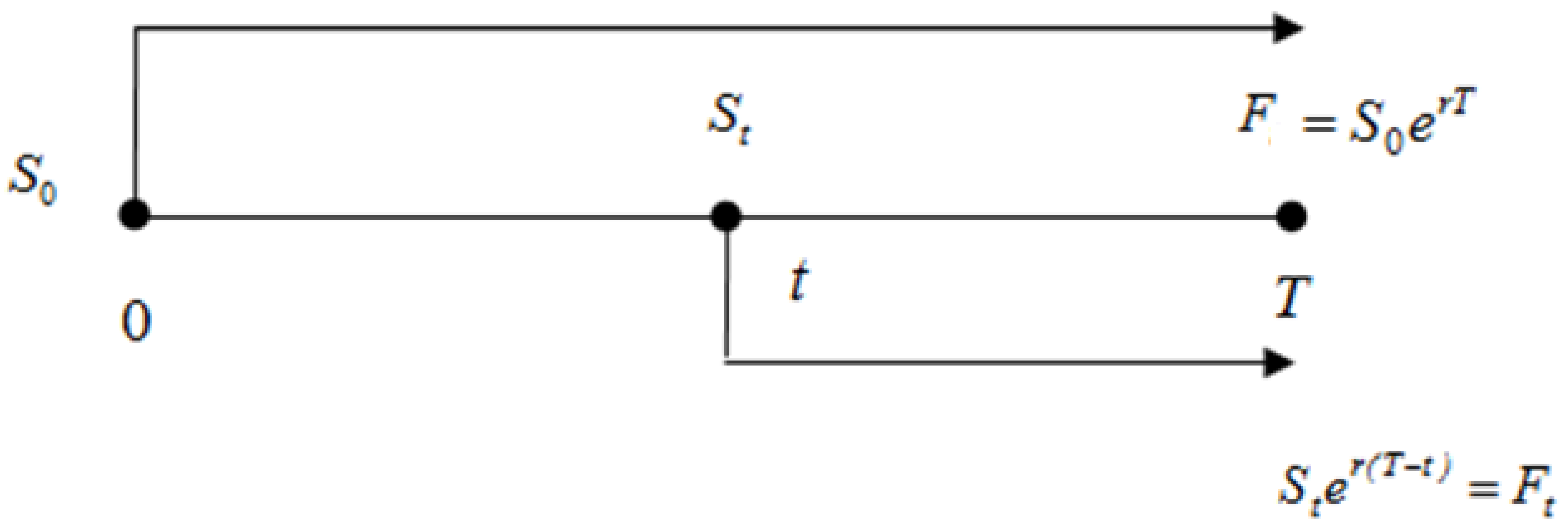
- The underlying asset pays predictable income

$$f_t = (S_t - D^*) - F \cdot e^{-(T-t)\cdot r}$$

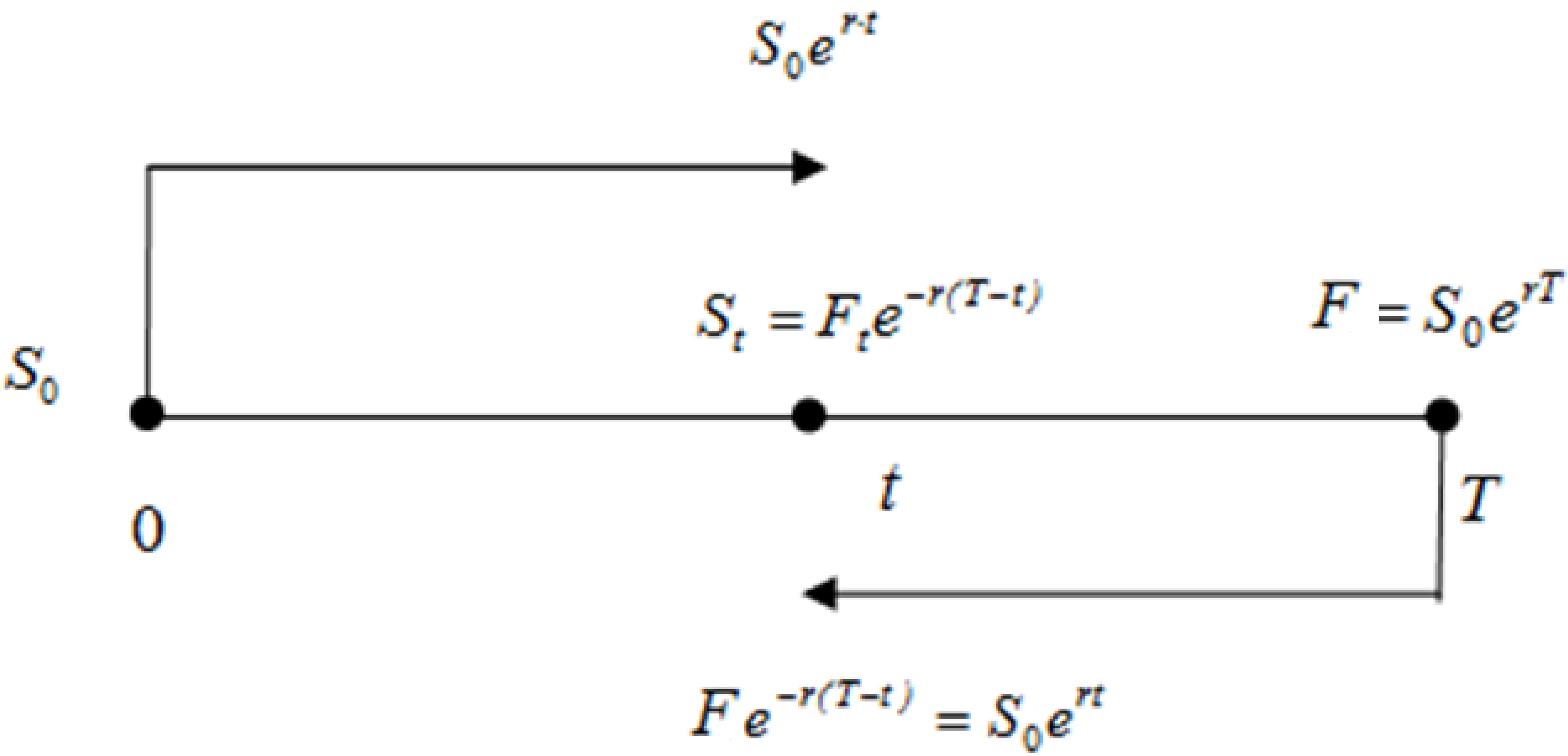
- The underlying asset pays continuous dividend yields

$$f_t = S_t e^{-(T-t)\cdot d} - F \cdot e^{-(T-t)\cdot r}$$

The underlying asset pays no income



The underlying asset pays no income



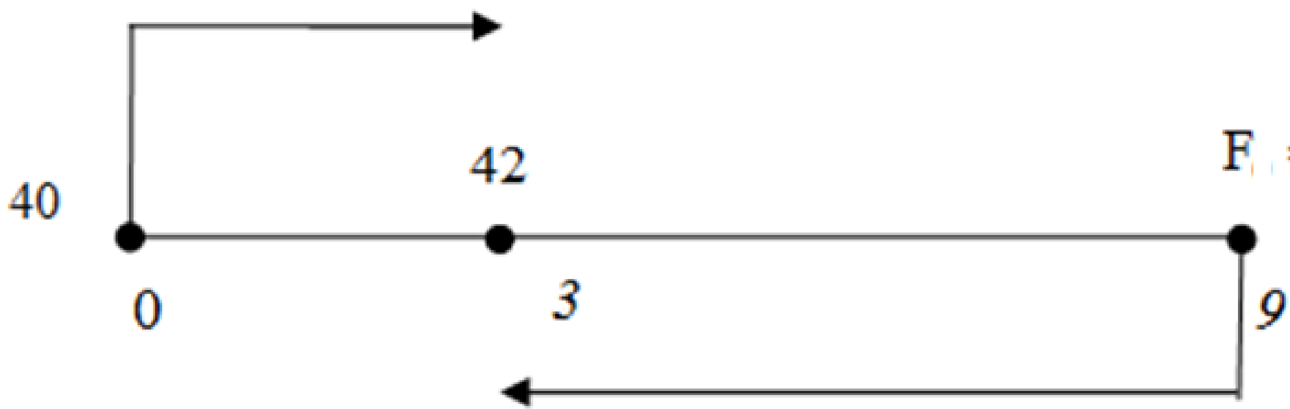
$$f_t = S_t - F e^{-r(T-t)} \qquad f_t = S_t - S_0 e^{rt} \qquad f_t = (F_t - F)e^{-r(T-t)}$$

Example

$$f_t = S_t - Fe^{-r(T-t)} \qquad f_t = S_t - S_0 e^{rt} \qquad f_t = (F_t - F)e^{-r(T-t)}$$

$$40 \cdot e^{0.01 \cdot 3} = 41.22 \qquad F_t = 42 \cdot e^{0.01 \cdot 6} = 44.597$$

$$F = 40 \cdot e^{0.01 \cdot 9} = 43.77$$

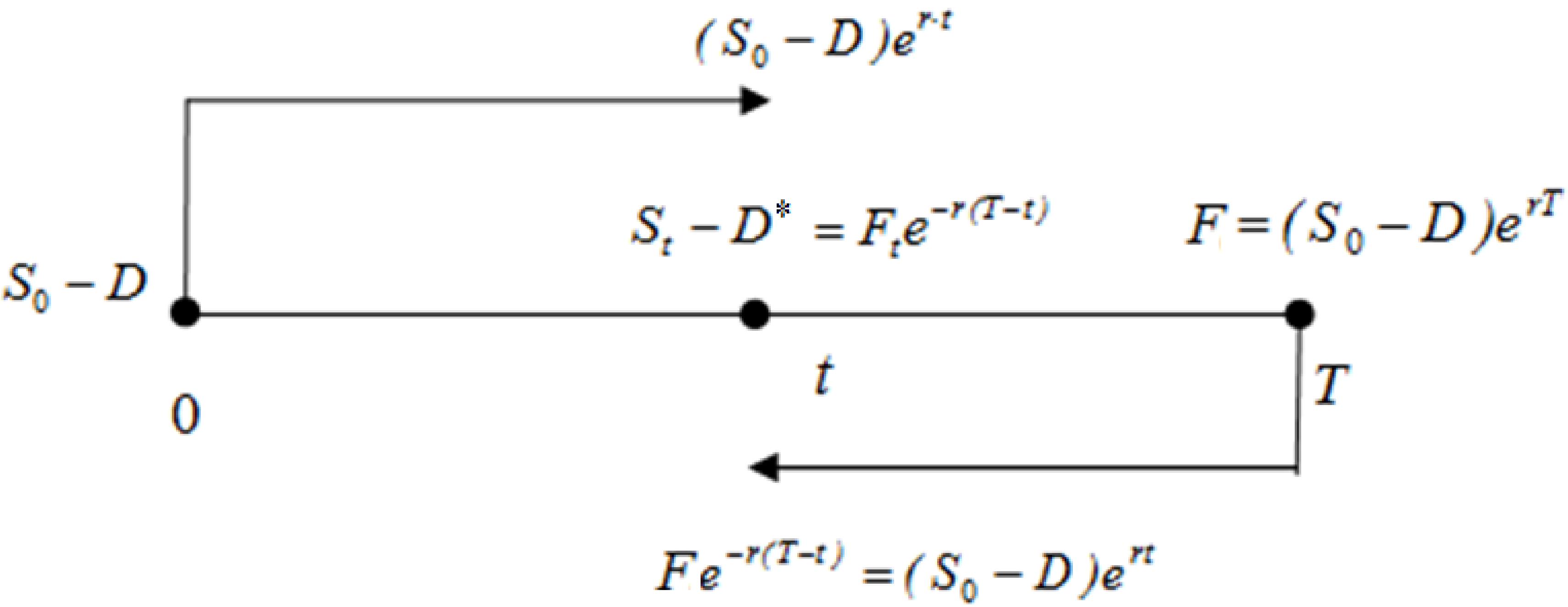


$$f_t = 0.78$$

$$Ke^{-r(T-t)} = 43.77 \cdot e^{-0.01 \cdot 6} = 41.22$$

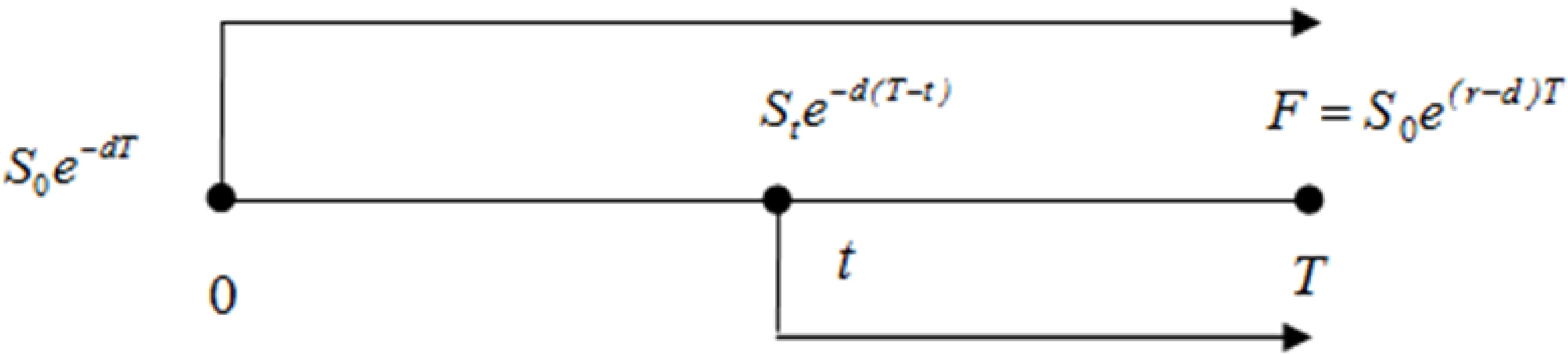
Interest rate 12%

The underlying asset pays predictable income



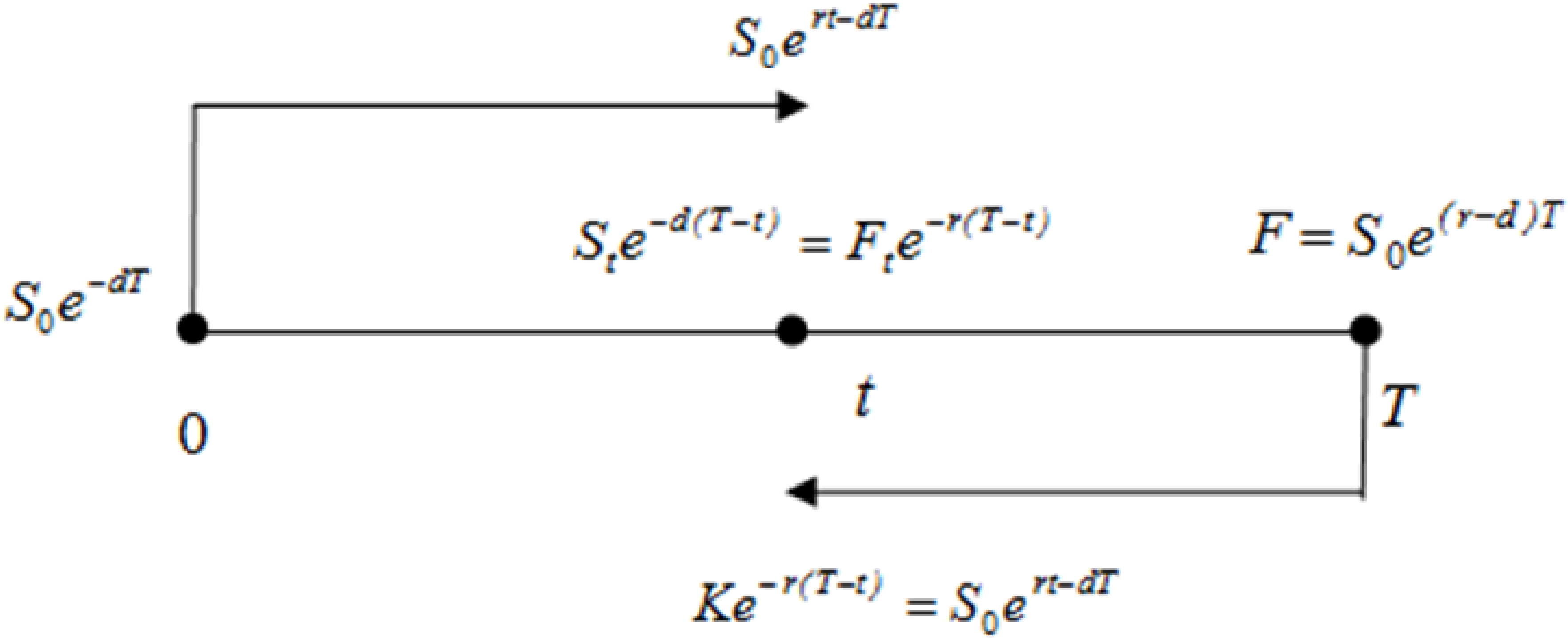
$f_t = (S_t - D^*) - Fe^{-r(T-t)}$ $\quad f_t = (S_t - D^*) - (S_0 - D)e^{rt}$ $\quad f_t = (F_t - F)e^{-r(T-t)}$

The underlying asset pays continuous dividend yields



$$F_t = S_t e^{-d(T-t)} e^{r(T-t)} = S_t e^{(r-d)(T-t)}$$

The underlying asset pays continuous dividend yields

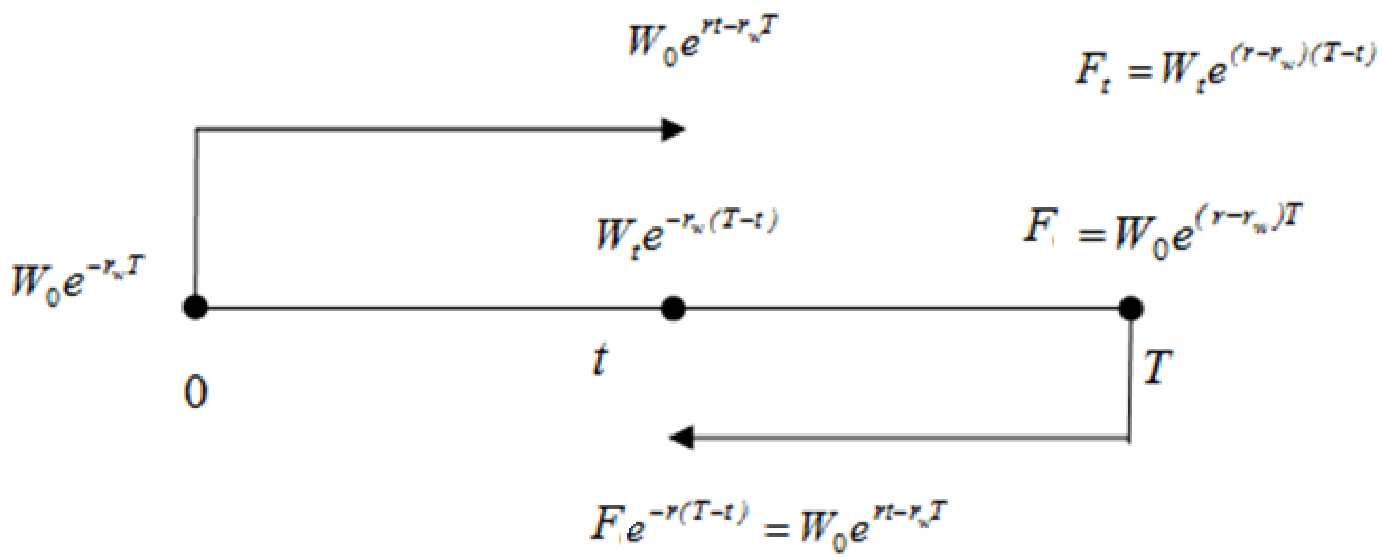


$f_t = S_t e^{-d(T-t)} - F e^{-r(T-t)}$ $\quad$ $f_t = S_t e^{-d(T-t)} - S_0 e^{rt-dT}$ $\quad$ $f_t = (F_t - F)e^{-r(T-t)}$

Forward contract on currencies

- W denotes the domestic/foreign exchange rate
- r – domestic riskless interest rate
- r_w – foreign riskless interest rate

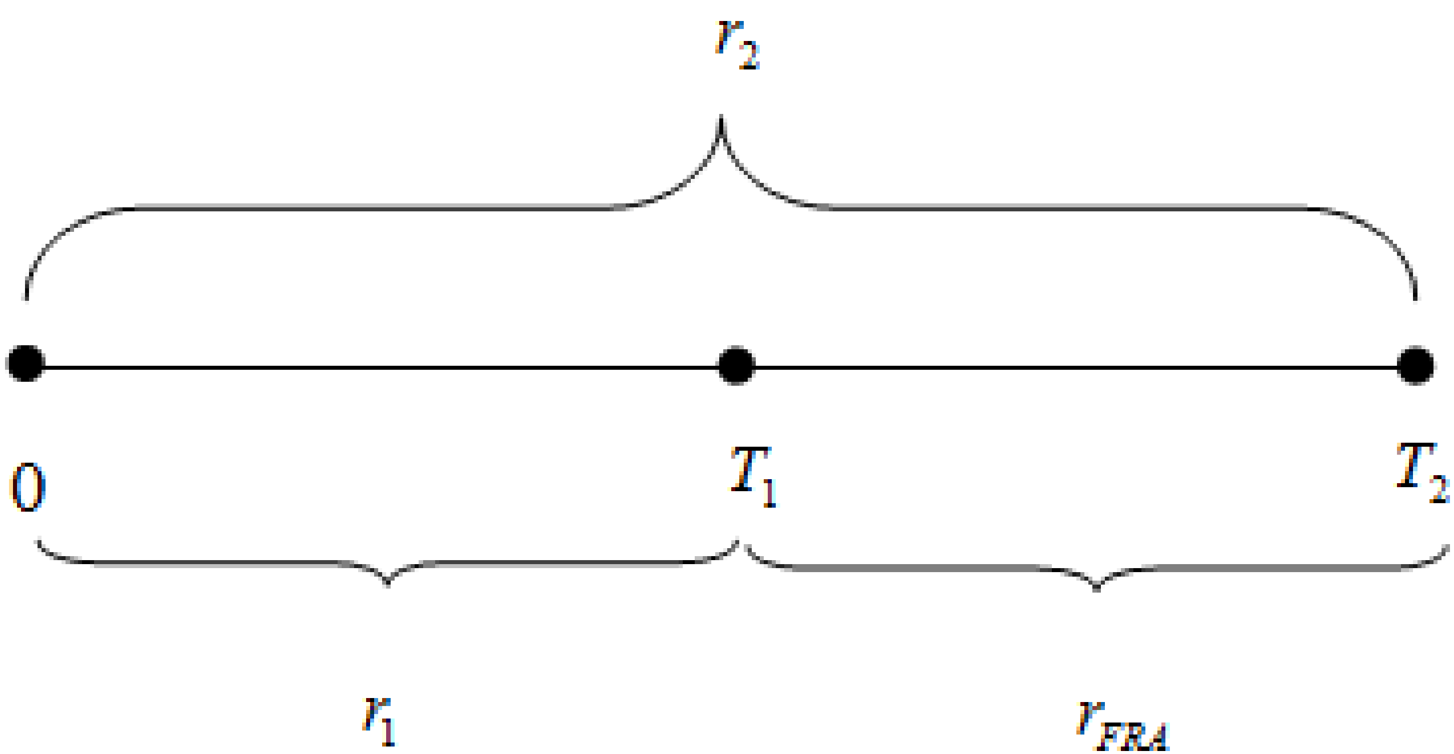
Forward contract on currencies



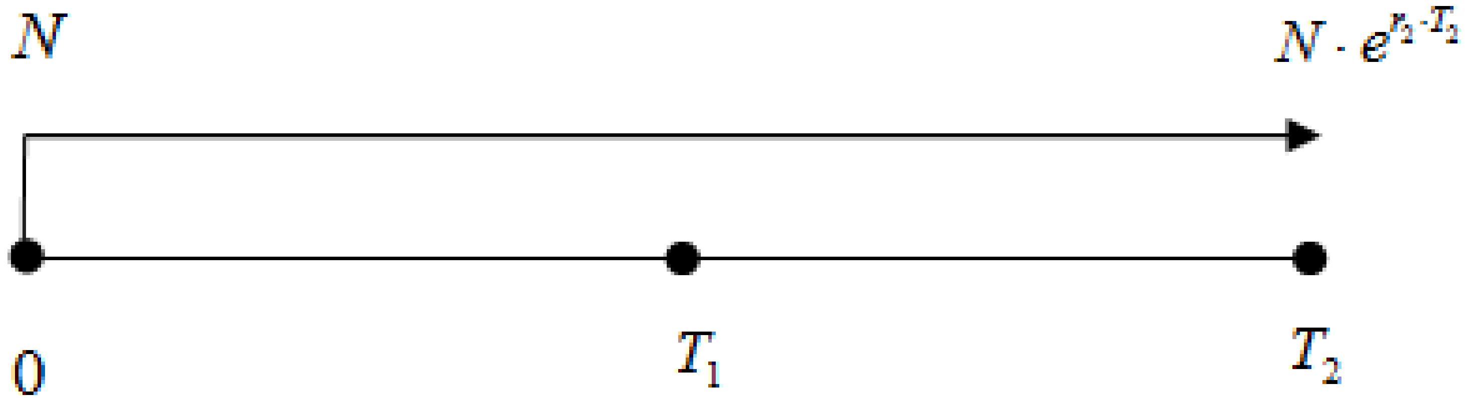
$f_t = W_t e^{-r_w (T-t)} - F e^{-r(T-t)}$ $\quad$ $f_t = W_t e^{-r_w (T-t)} - W_0 e^{rt - r_w T}$ $\quad$ $f_t = (F_t - F) e^{-r(T-t)}$

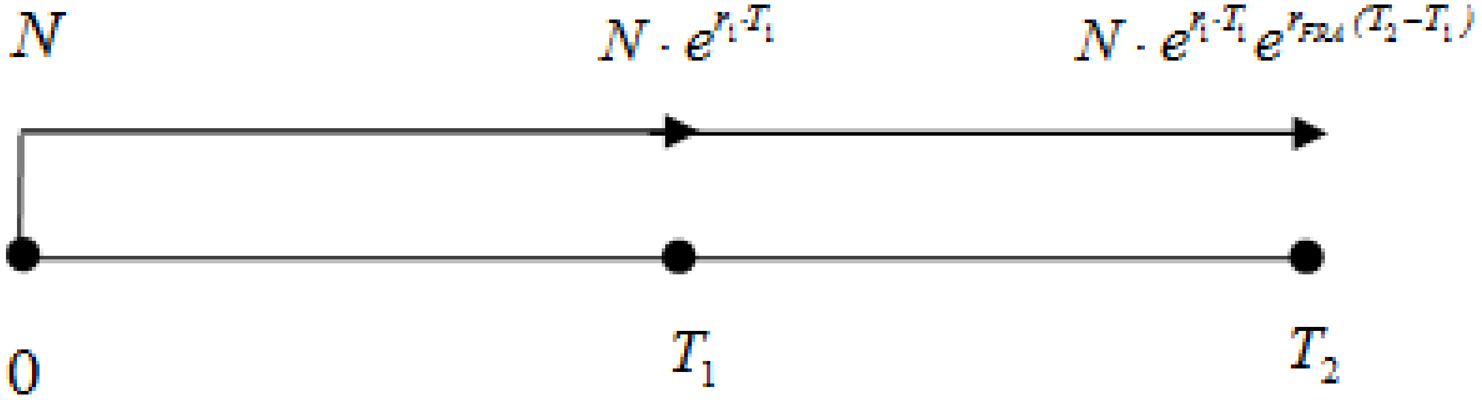
Forward Rate Agreements (FRA)

FRA – a forward contract where it is agreed that a certain interest rate r_{FRA} will apply to a certain principal to a specified future time period



FRA pricing – continuously compounded interest rate





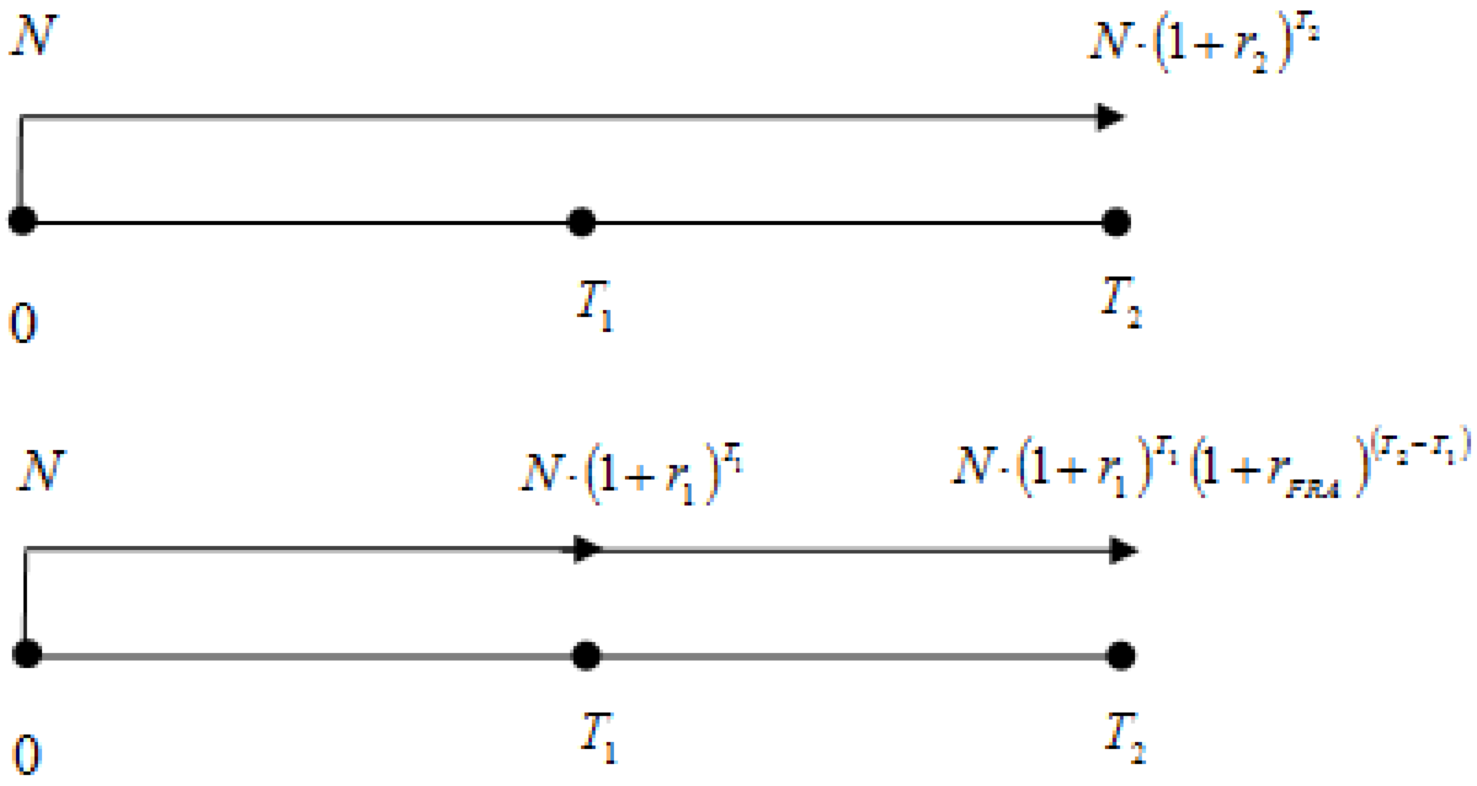
FRA pricing

$$N \cdot e^{r_1 \cdot T_1} e^{r_{FRA}(T_2 - T_1)} = N \cdot e^{r_2 \cdot T_2}$$

$$r_1 \cdot T_1 + r_{FRA}(T_2 - T_1) = r_2 \cdot T_2$$

$$r_{FRA} = \frac{r_2 \cdot T_2 - r_1 \cdot T_1}{T_2 - T_1}$$

FRA pricing – compound interest rate

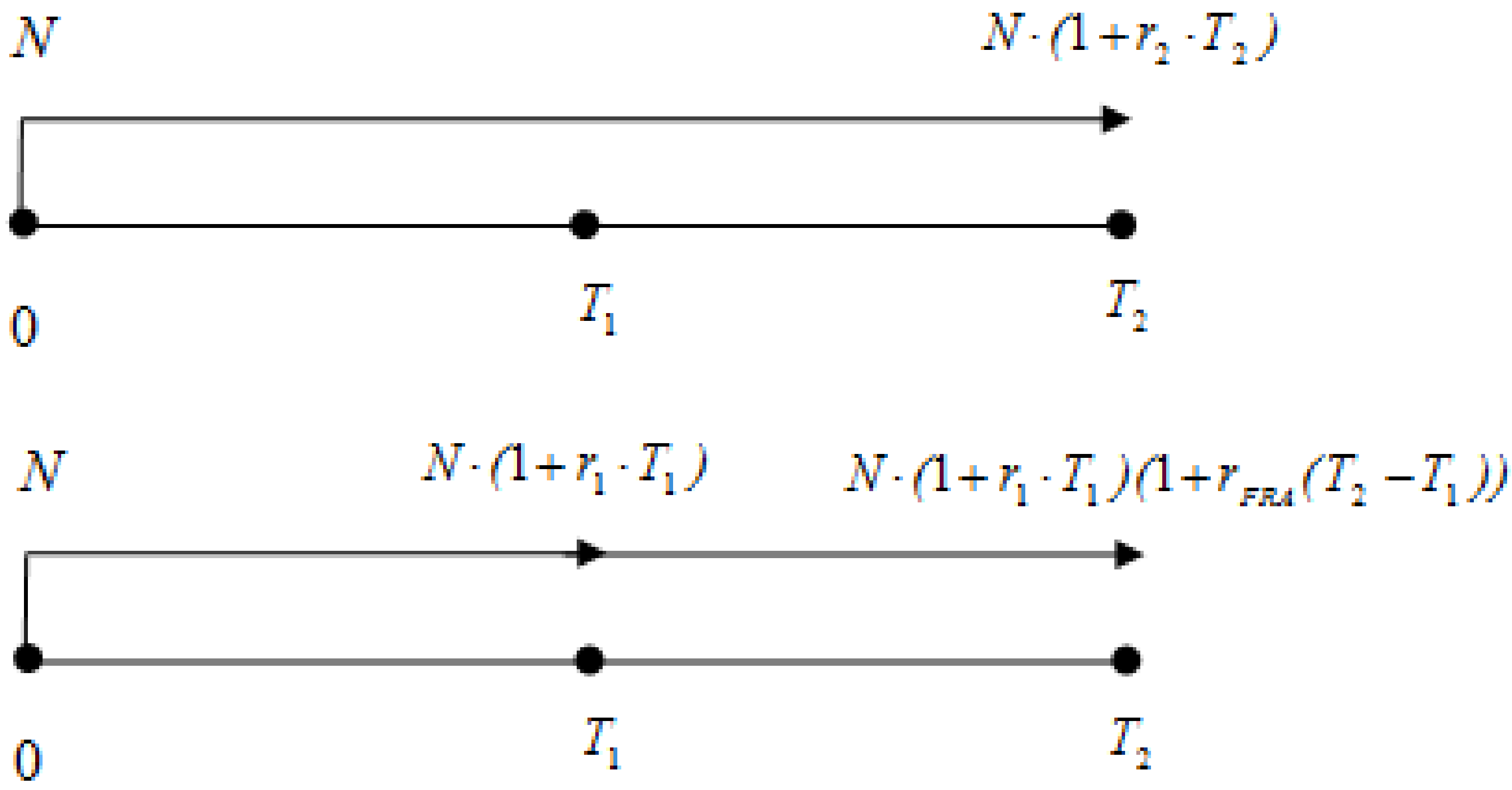


FRA pricing

$$N(1+r_2)^{T_2} = N(1+r_1)^{T_1}(1+r_{FRA})^{T_2-T_1}$$

$$r_{FRA} = \sqrt[T_2-T_1]{(1+r_2)^{T_2} \big/ (1+r_1)^{T_1}} - 1$$

FRA pricing – simple interest rate



FRA pricing

$$N \cdot (1 + r_1 \cdot T_1)(1 + r_{FRA}(T_2 - T_1)) = N \cdot (1 + r_2 \cdot T_2)$$

$$1 + r_{FRA}(T_2 - T_1) + r_1 \cdot T_1 + r_1 \cdot T_1 \cdot r_{FRA}(T_2 - T_1) = 1 + r_2 \cdot T_2$$

$$r_{FRA}(T_2 - T_1) \cdot (1 + r_1 \cdot T_1) = r_2 \cdot T_2 - r_1 \cdot T_1$$

$$r_{FRA} = \frac{1}{T_2 - T_1} \frac{r_2 \cdot T_2 - r_1 \cdot T_1}{1 + r_1 \cdot T_1}$$

Example

- Bank offers the FRA 3x3 rate of 5.8% per annum, the reference rate for 3 months is 5.0%, and for 6 months is 5.5%. Is this rate a fair rate?

$$T_1 = 0.25 \quad T_2 = 0.5 \quad r_1 = 5.0\% \quad r_2 = 5.5\%$$

$$r_{FRA} = \frac{1}{T_2 - T_1} \frac{r_2 T_2 - r_1 T_1}{1 + r_1 T_1}$$

$$r_{FRA} = \frac{1}{0.5 - 0.25} \frac{0.055 \cdot 0.5 - 0.05 \cdot 0.25}{1 + 0.05 \cdot 0.25} = 0.059$$

- $FRA \quad m \times n$

The binomial option pricing model

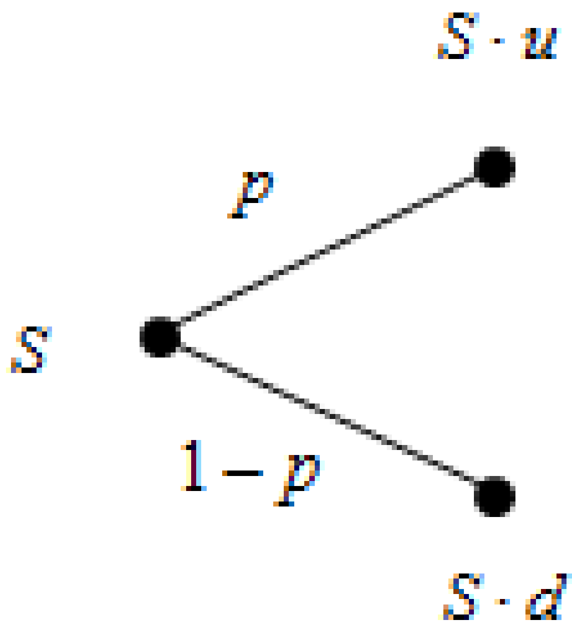
- Option valuation is a step process:

1. Create the binomial price tree (lattice), for a number of time steps between the valuation and expiration dates. Each node in the lattice represents a possible price of the underlying asset at a given point in time.

2. Create the binomial option tree

- ➢ Find option value at each final node.
- ➢ Find option value at earlier nodes.

One-period binomial price tree



- At each step, it is assumed that the price of the underlying instrument will move up or down by a specific factor (*u* or *d*) per step of the tree.

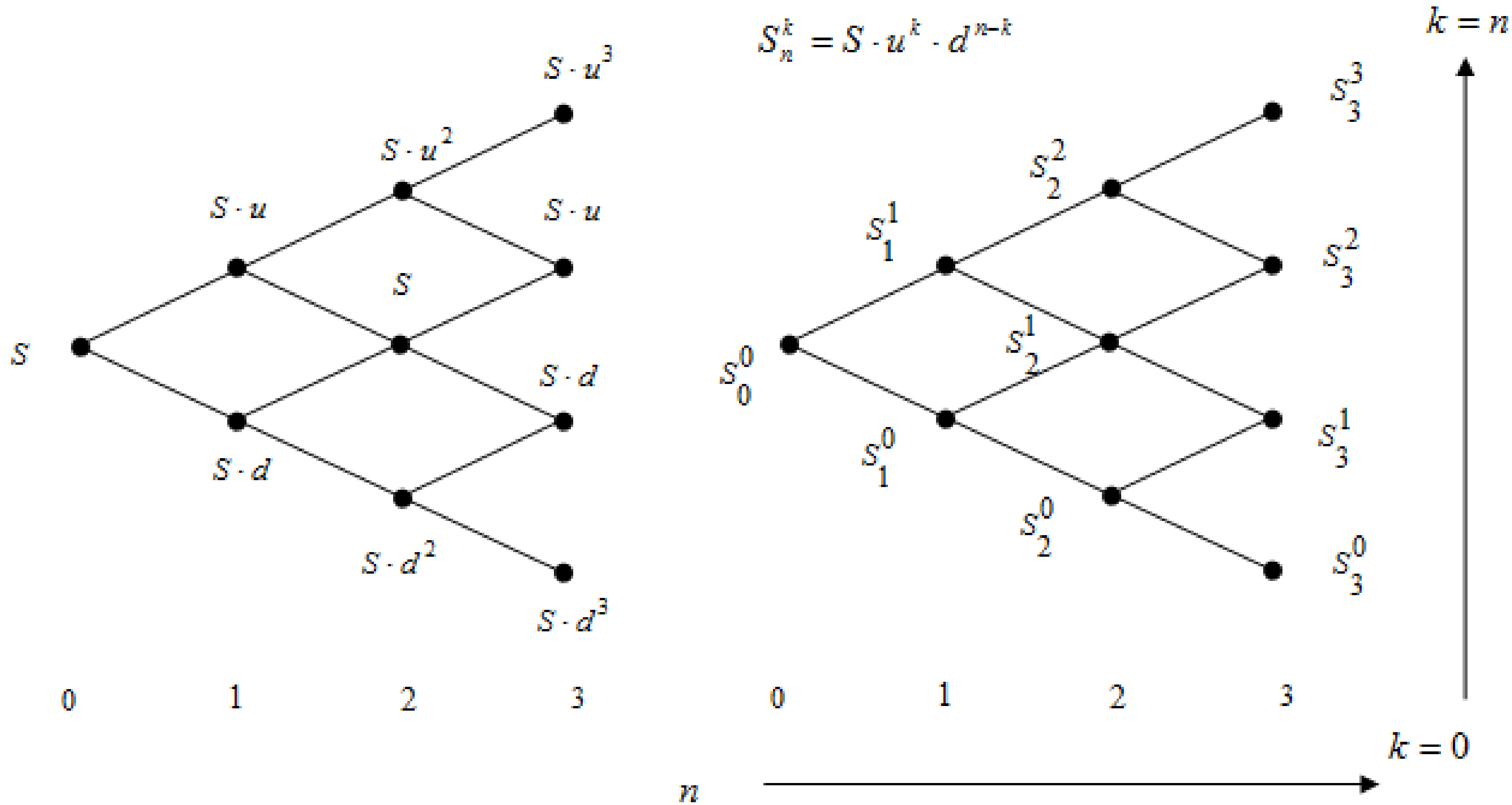
p – the probability

$$p \in (0,1)$$

- By definition

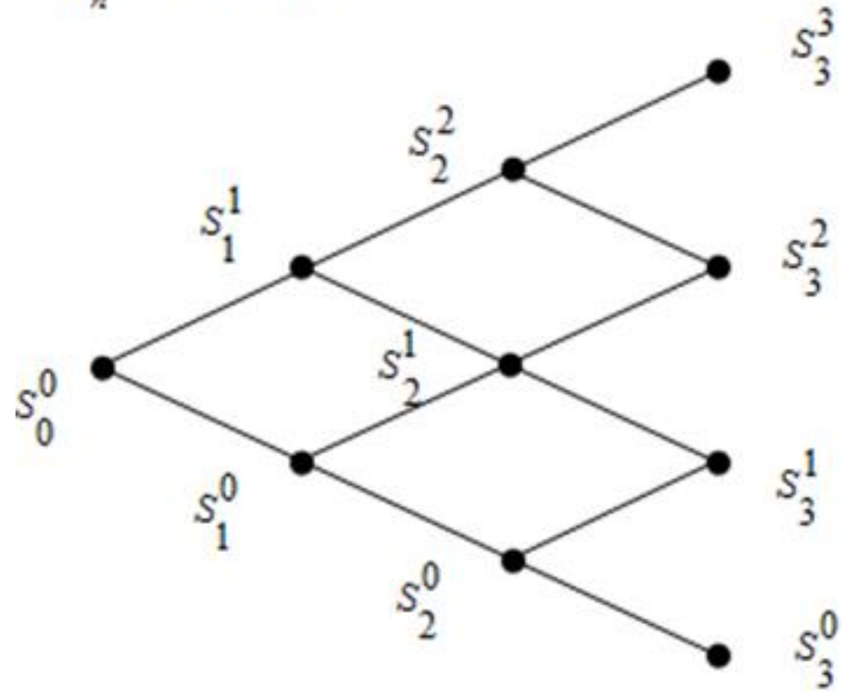
$$u > 1 \qquad d \in (0,1) \qquad d = \frac{1}{u}$$

The binomial price tree



$$\begin{array}{cccc}
S_0^0 & S_1^1 & S_2^2 & S_3^3 \\
 & S_1^0 & S_2^1 & S_3^2 \\
 & & S_2^0 & S_3^1 \\
 & & & S_3^0
\end{array}$$

$$S_n^k = S \cdot u^k \cdot d^{n-k}$$



The binomial price tree

- At node (n,k) $k = 0,1,\ldots,n$

Binomial coefficient

with probability

$$\binom{n}{k} \cdot p^k (1-p)^{n-k}$$

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!}$$

$n!$ - factorial of n

the price is $S_n^k = S \cdot u^k \cdot d^{n-k}$

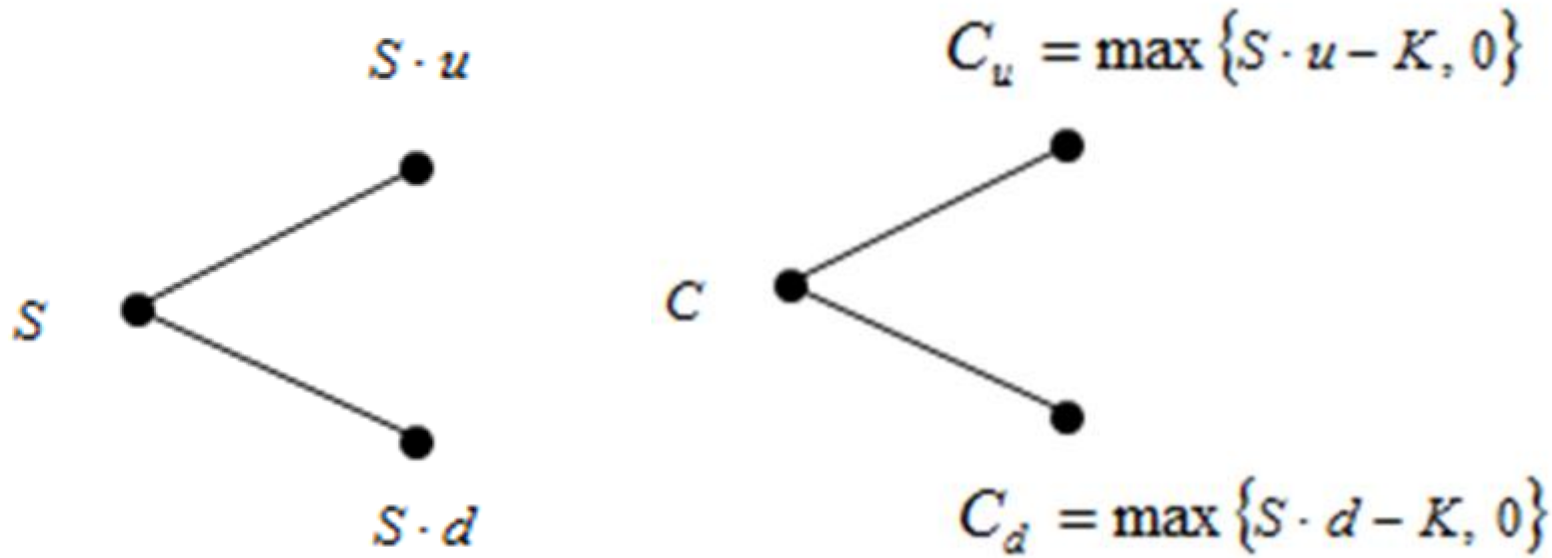
$$S_2^1 = S \cdot u \cdot d = S$$

$$\binom{2}{1} \cdot p^1 (1-p)^{2-1} = 2 \cdot p \cdot (1-p)$$

$$S_3^1 = S \cdot u \cdot d^2 = S \cdot d$$

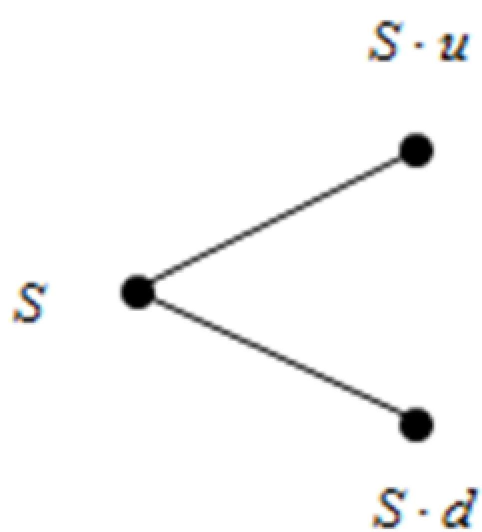
$$3 \cdot p \cdot (1-p)^2$$

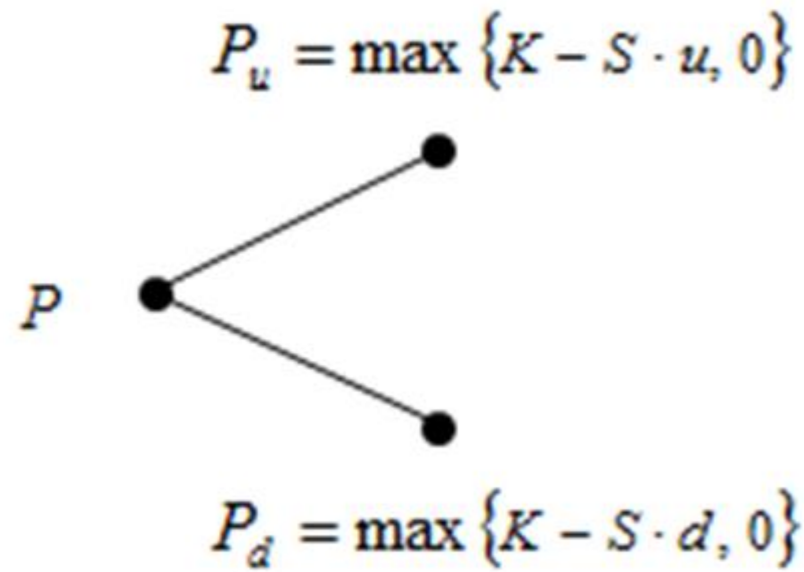
One-period binomial option tree (a call option)



K is the strike price

One-period binomial option tree (a put option)





K is the strike price

Replicating portfolio (call option, compound interest)

- Option can be replicated by portfolio of other securities
- x – the underlying asset
- b – bonds

$$C = x + b$$

$u \cdot x + (1 + r) \cdot b$ if the price of the underlying asset will move up

$d \cdot x + (1 + r) \cdot b$ if the price of the underlying asset will move down

$$u \cdot x + R \cdot b = C_u$$

$$d \cdot x + R \cdot b = C_d$$

$$R = 1 + r$$

$$x = \frac{C_u - C_d}{u - d}$$

$$b = \frac{u \cdot C_d - d \cdot C_u}{R(u - d)}$$

$$C = x + b = \frac{1}{R}\left(\frac{R - d}{u - d} C_u + \frac{u - R}{u - d} C_d\right)$$

$$\boxed{C = \frac{1}{R}\left(q \cdot C_u + (1 - q) \cdot C_d\right)}$$

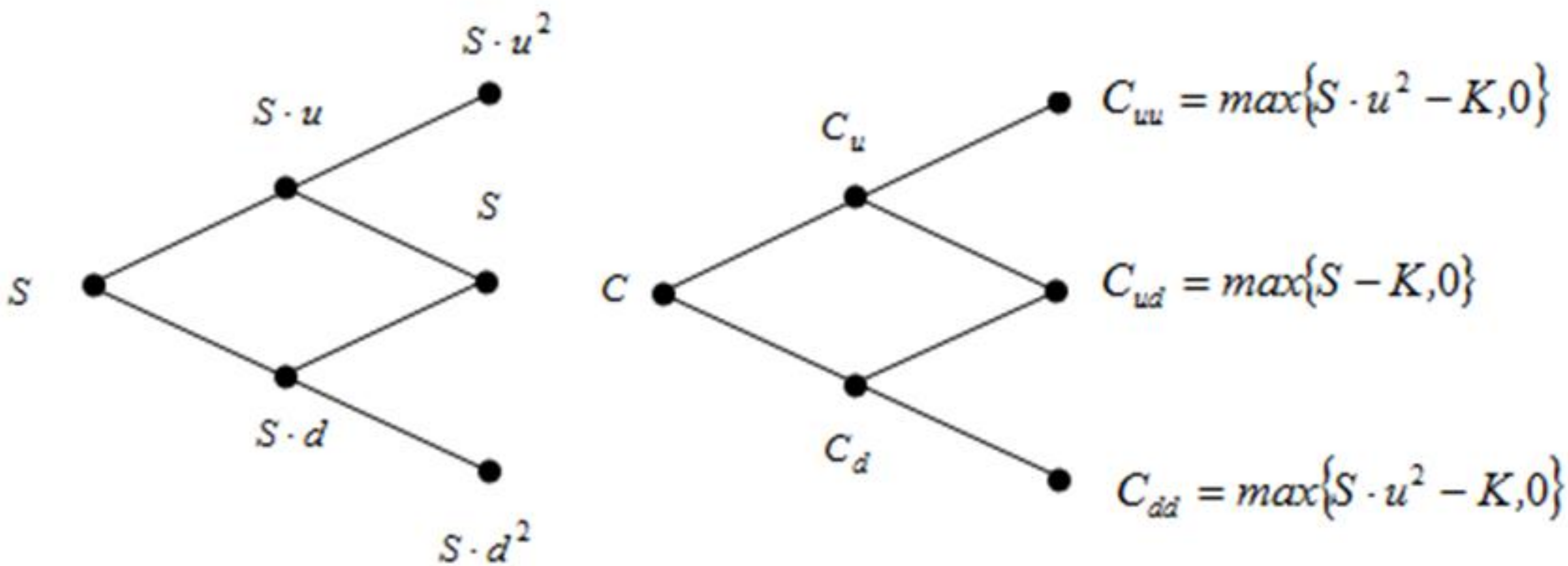
$$q = \frac{R - d}{u - d}$$

$$u = e^{\sigma \cdot \sqrt{\Delta t}}$$

$$d = e^{-\sigma \cdot \sqrt{\Delta t}} = 1/u$$

Two-period binomial price and option trees

(the underlying asset does not pay dividends)



$$C_u = \frac{1}{R}\left(q \cdot C_{uu} + (1-q) \cdot C_{ud}\right) \qquad C_d = \frac{1}{R}\left(q \cdot C_{ud} + (1-q) \cdot C_{dd}\right)$$

$$C = \frac{1}{R}\left(q \cdot C_u + (1-q) \cdot C_d\right) \qquad q = \frac{R-d}{u-d}$$

$$C = \frac{1}{R}\left(q \cdot \frac{1}{R}\left(q \cdot C_{uu} + (1-q) \cdot C_{ud}\right) + (1-q) \cdot \frac{1}{R}\left(q \cdot C_{ud} + (1-q) \cdot C_{dd}\right)\right)$$

$$C = \frac{1}{R^2}\left(q^2 \cdot C_{uu} + 2q \cdot (1-q) \cdot C_{ud} + (1-q)^2 \cdot C_{dd}\right)$$

Replicating portfolio (call option, continuously compounded interest)

$$u \cdot x + b \cdot e^{r \cdot \Delta t} = C_u$$

$$x = \frac{C_u - C_d}{u - d}$$

$$d \cdot x + b \cdot e^{r \cdot \Delta t} = C_d$$

$$b = e^{-r \cdot \Delta t}\left(\frac{u \cdot C_d - d \cdot C_u}{u - d}\right)$$

$$C = x + b = e^{-r \cdot \Delta t}\left(\frac{e^{r \cdot \Delta t} - d}{u - d} C_u + \frac{u - e^{r \cdot \Delta t}}{u - d} C_d\right)$$

$$C = e^{-r \cdot \Delta t}\left(q \cdot C_u + (1 - q) \cdot C_d\right)$$

$$q = \frac{e^{r \cdot \Delta t} - d}{u - d}$$

The binomial option pricing model

- Compound interest

$$C = \frac{1}{R}\left(q \cdot C_u + (1-q) \cdot C_d\right)$$

$$R = 1 + r$$

$$q = \frac{R-d}{u-d}$$

- Continuously compounded interest

$$C = e^{-r \cdot \Delta t}\left(q \cdot C_u + (1-q) \cdot C_d\right)$$

$$q = \frac{e^{r \cdot \Delta t} - d}{u-d}$$

$$u = e^{\sigma \cdot \sqrt{\Delta t}} \qquad d = e^{-\sigma \cdot \sqrt{\Delta t}} = 1/u$$

Example

$$S = 50 \qquad K = 48 \qquad \sigma = 0.2 \qquad r = 0.1$$

Expiration time – 3 months

$$u = e^{0.2\cdot\sqrt{1/12}} = 1.059434$$

$$u = e^{\sigma\cdot\sqrt{\Delta t}}$$

$$d = 1/u = 0.9439$$

$$d = e^{-\sigma\cdot\sqrt{\Delta t}} = 1/u$$

$$R = 1 + 0.1/12 = 1.008333$$

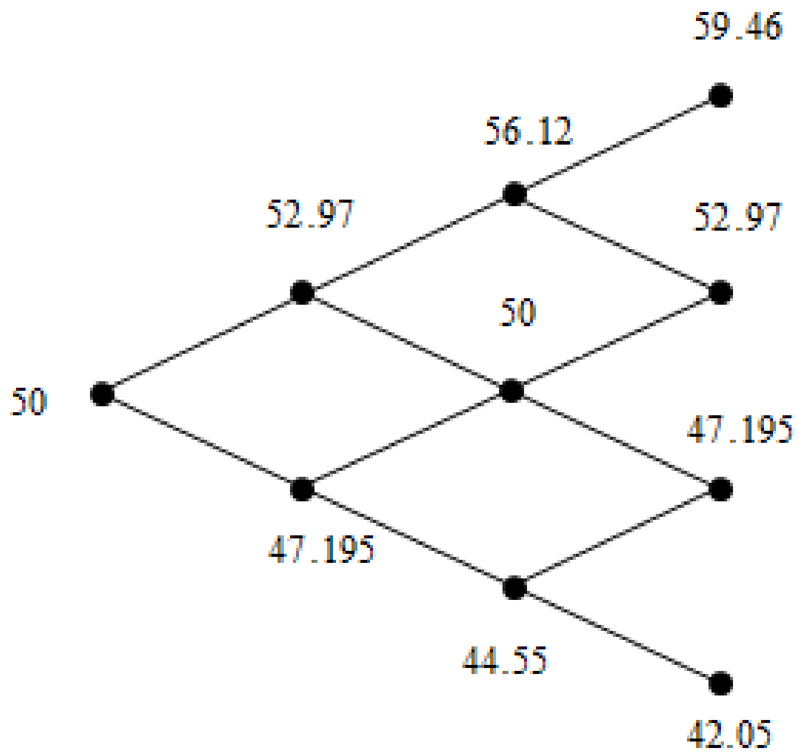
$$R = 1 + r$$

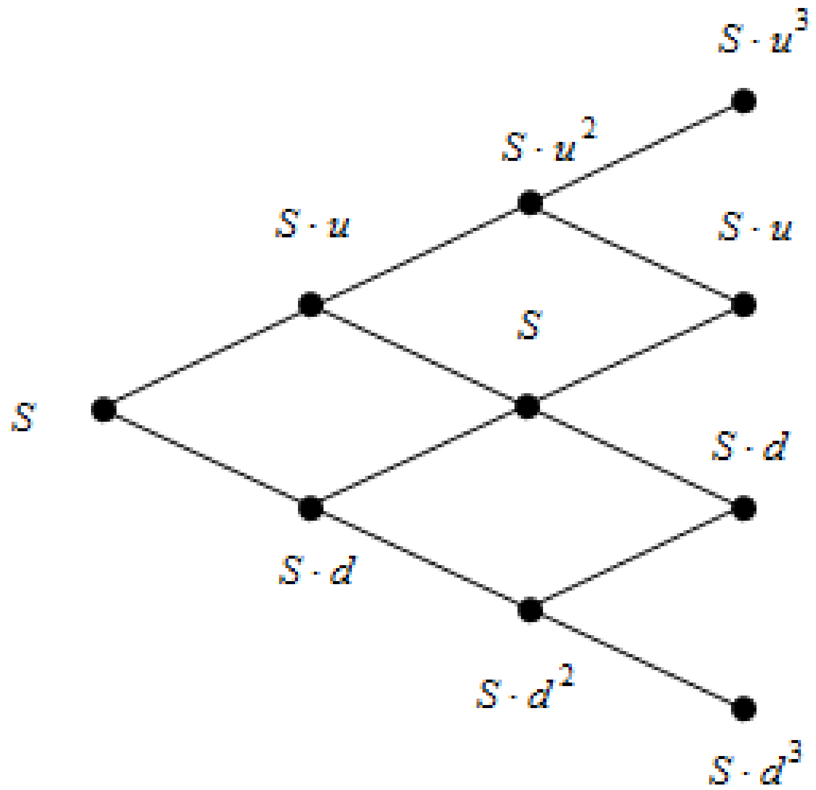
$$q = \frac{1.0083 - 0.9439}{1.05943 - 0.9439} = 0.557699$$

$$q = \frac{R-d}{u-d}$$

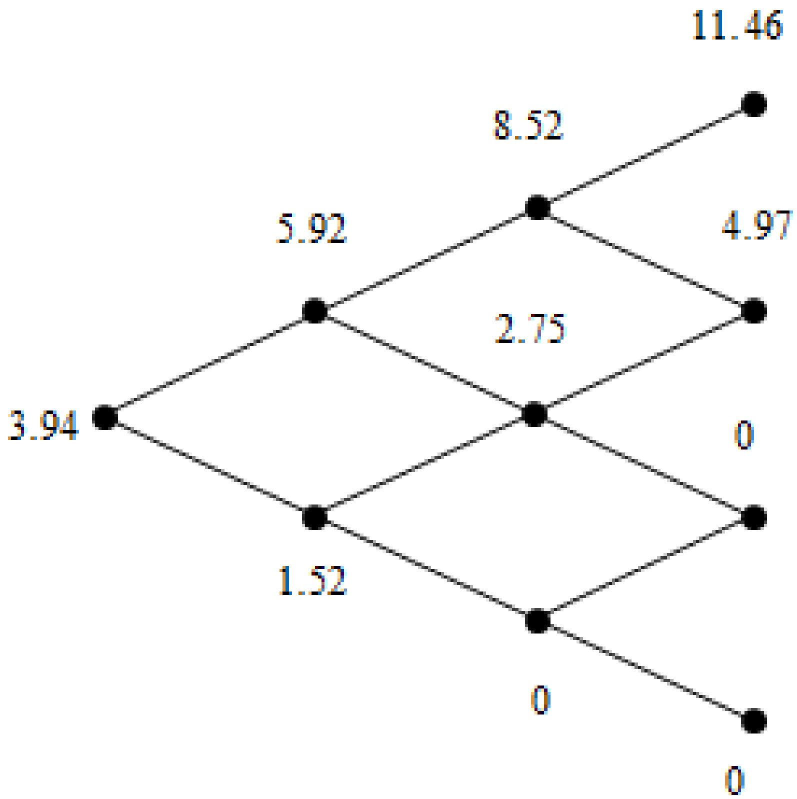
$$1 - q = 0.442301$$

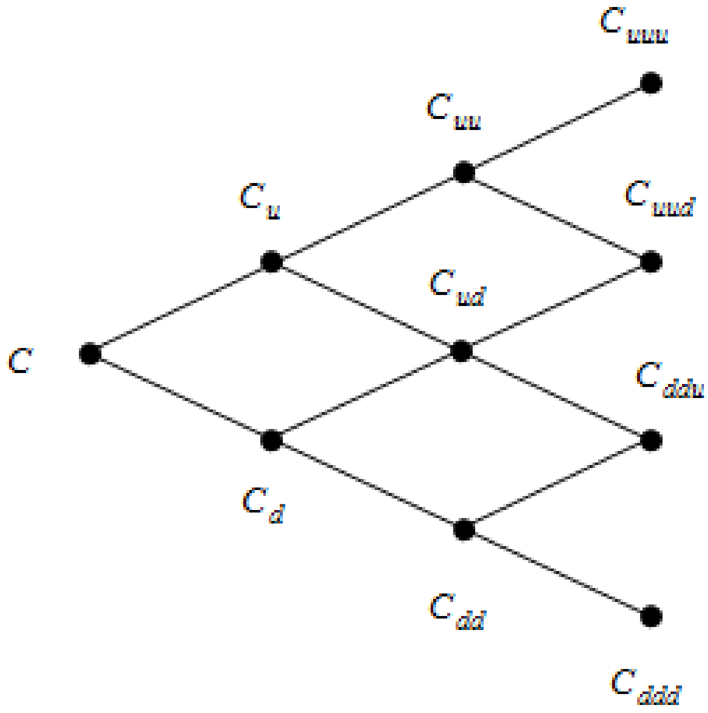
Three-period binomial price tree





Three-period binomial option tree (European call option)



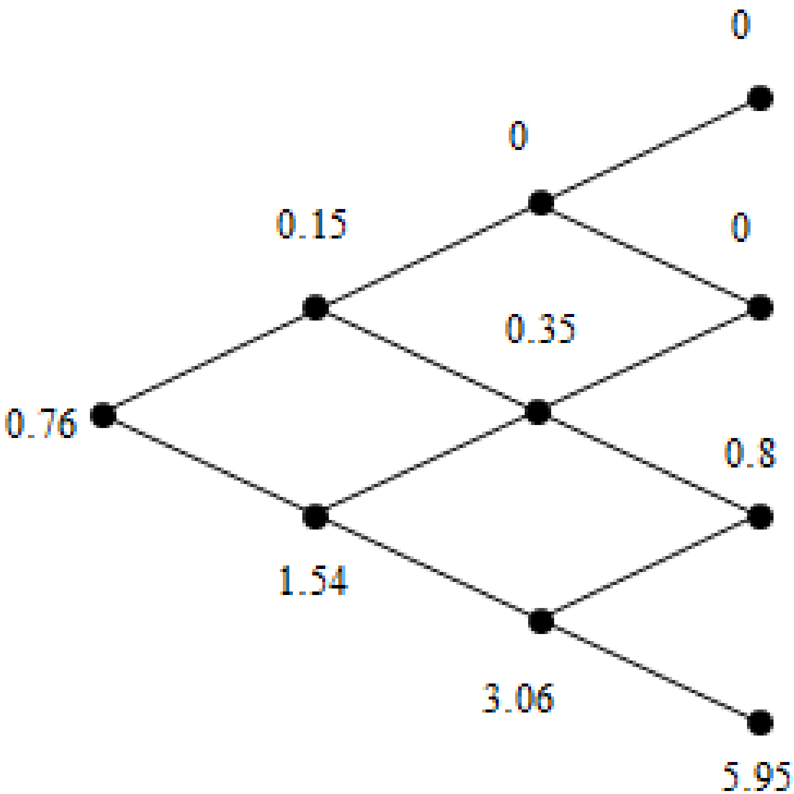


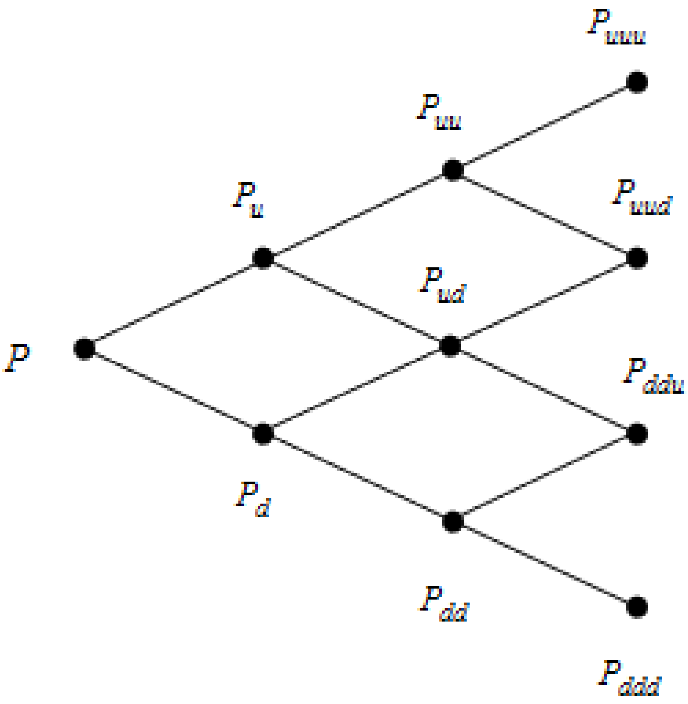
$C_{uuu} = \max\{S \cdot u^3 - K, 0\}$, $C_{uud} = \max\{S \cdot u - K, 0\}$,

$C_{ddu} = \max\{S \cdot d - K, 0\}$, $C_{ddd} = \max\{S \cdot d^3 - K, 0\}$,

$$C_{uu} = \frac{1}{R}\left(q \cdot C_{uuu} + (1-q) \cdot C_{uud}\right), \quad C_u = \frac{1}{R}\left(q \cdot C_{uu} + (1-q) \cdot C_{ud}\right)$$

Three-period binomial option tree (European put option)



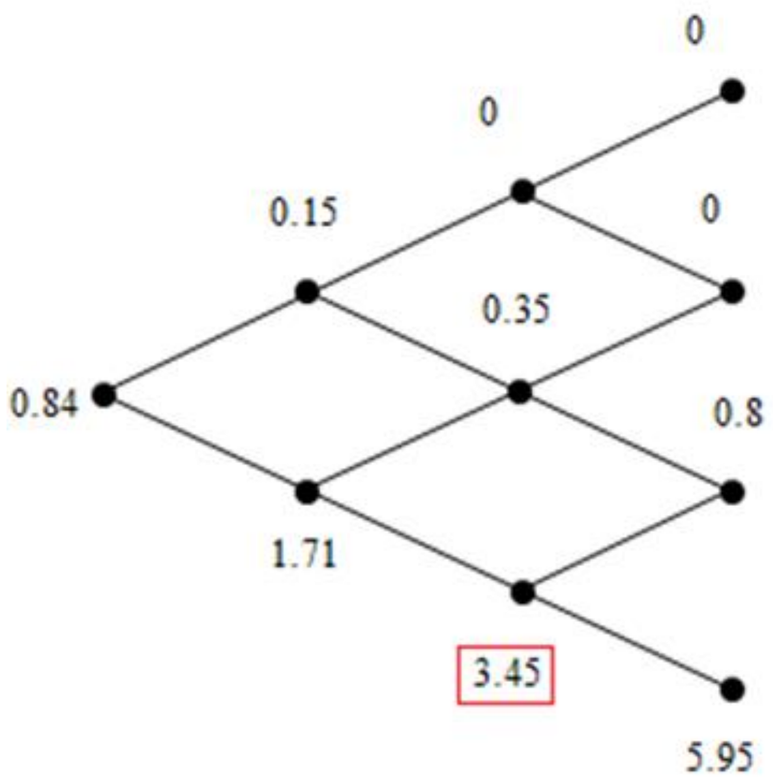


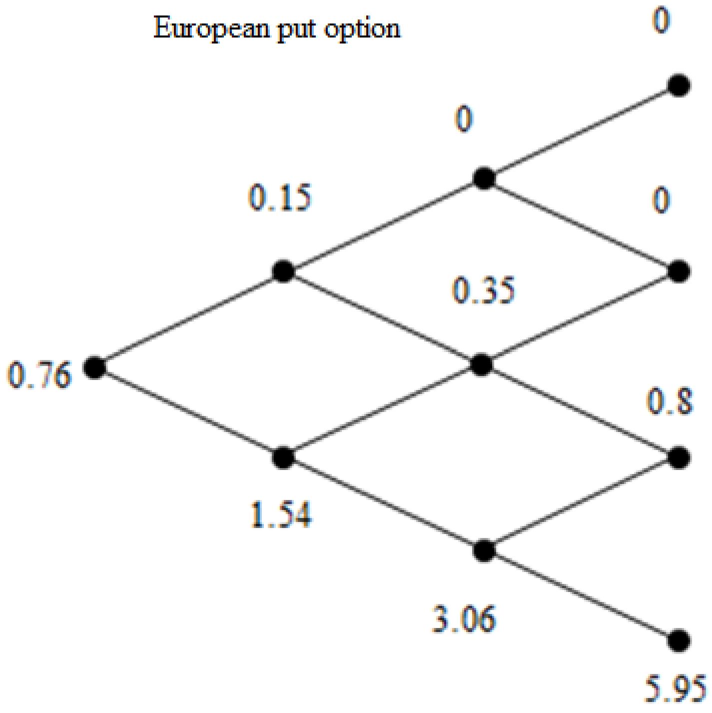
$P_{uuu} = max\{K - S \cdot u^3, 0\}$, $P_{uud} = max\{K - S \cdot u, 0\}$,

$P_{ddu} = max\{K - S \cdot d, 0\}$, $P_{ddd} = max\{K - S \cdot d^3, 0\}$,

$$P_{uu} = \frac{1}{R}(q \cdot P_{uuu} + (1-q) \cdot P_{uud}), \quad P_u = \frac{1}{R}(q \cdot P_{uu} + (1-q) \cdot P_{ud})$$

Three-period binomial option tree (American put option)





$$P_{dd} = K - S_{dd} = 48 - 44.55 = 3.45$$

Option	Compound interest	Continuously compounded interest
European call option	3.9396	3.9431
European put option	0.7593	0.7580
American put option	0.8357	0.8345

Example

- Create a three-period binominal price tree and find the fair value of an European call and put options and an American put option on a nondividend-paying stock if the initial stock price is 62 PLN, the compound risk-free interest rate is 12% per annum, the stock volatility is 20%, the strike price of 60 PLN is expiring at the end of the third month (at the end of the third week).

Period - a month	Period - a week
$u = e^{0.1\cdot\sqrt{1/12}} = 1.029288$	$u = e^{0.1\cdot\sqrt{1/52}} = 1.013964$
$d = 1/u = 0.971545$	$d = 1/u = 0.986228$
$R = 1 + 0.12/12 = 1.01$	$R = 1 + 0.12/52 = 1.002308$
$q = 0.665965$	$q = 0.579736$
$1 - q = 0.334035$	$1 - q = 0.420264$

Three months

Price tree

62.00	63.82	65.68	67.61
	60.24	62.00	63.82
		58.52	60.24
			56.86

European call option

3.88	5.00	6.28	7.61
	1.76	2.59	3.82
		0.16	0.24
			0.00

European put option

0.11	0.00	0.00	0.00
	0.34	0.00	0.00
		1.04	0.00
			3.14

American put option

0.16	0.00	0.00	0.00
	0.49	0.00	0.00
		1.48	0.00
			3.14

Period - three weeks

Price tree

62.00	62.87	63.74	64.63
	61.15	62.00	62.87
		60.30	61.15
			59.47

European call option

2.45	3.14	3.88	4.63
	1.51	2.14	2.87
		0.66	1.15
			0.00

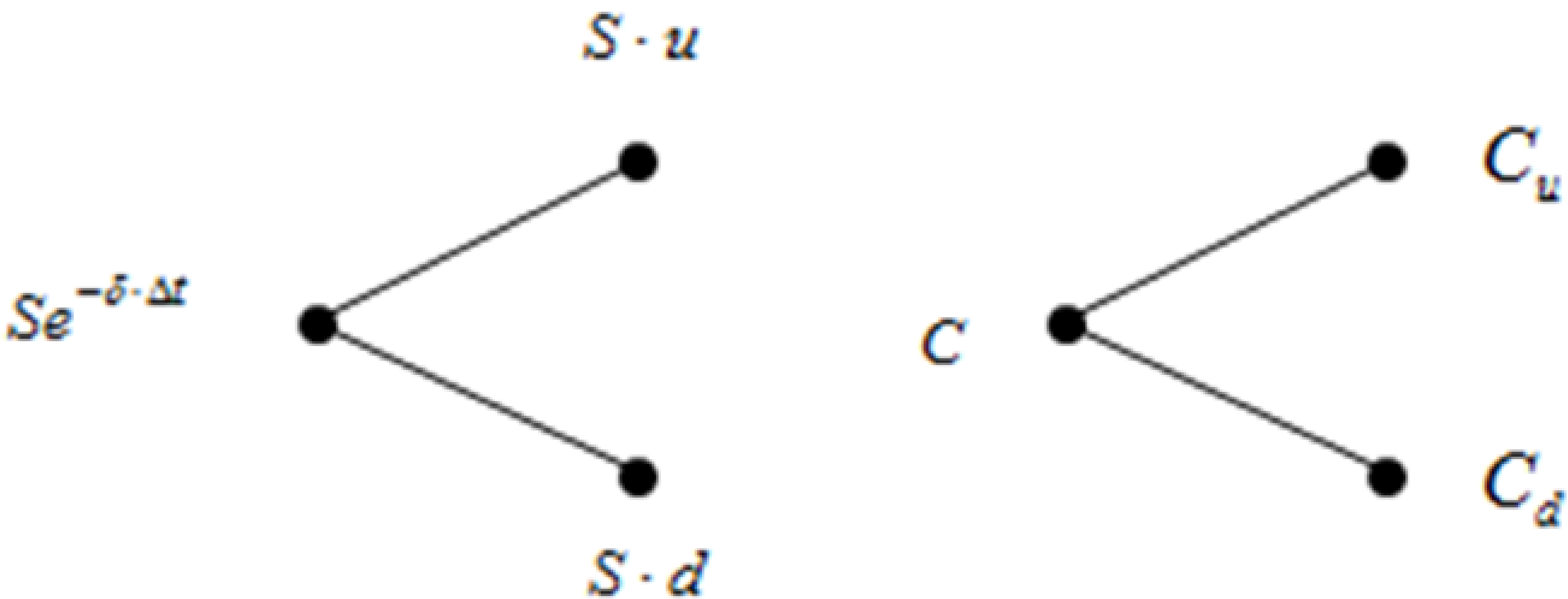
European put option

0.04	0.00	0.00	0.00
	0.09	0.00	0.00
		0.22	0.00
			0.53

American put option

0.04	0.00	0.00	0.00
	0.09	0.00	0.00
		0.22	0.00
			0.53

The underlying asset pays continuous dividend δ



$$u \cdot x + b \cdot e^{r \cdot \Delta t} = C_u$$

$$x = \frac{C_u - C_d}{u - d}$$

$$d \cdot x + b \cdot e^{r \cdot \Delta t} = C_d$$

$$b = e^{-r \cdot \Delta t} \left(\frac{u \cdot C_d - d \cdot C_u}{u - d} \right)$$

$$C = x \cdot e^{-\delta \cdot \Delta t} + b = \left(\frac{C_u - C_d}{u - d} \right) e^{-\delta \cdot \Delta t} + \left(\frac{u \cdot C_d - d \cdot C_u}{u - d} \right) e^{-r \cdot \Delta t}$$

$$C = x \cdot e^{-\delta \cdot \Delta t} + b = \left(\frac{C_u - C_d}{u - d} \right) e^{-\delta \cdot \Delta t} + \left(\frac{u \cdot C_d - d \cdot C_u}{u - d} \right) e^{-r \cdot \Delta t}$$

$$C = e^{-r \cdot \Delta t} \left[\left(\frac{C_u - C_d}{u - d} \right) e^{(r - \delta) \cdot \Delta t} + \left(\frac{u \cdot C_d - d \cdot C_u}{u - d} \right) \right]$$

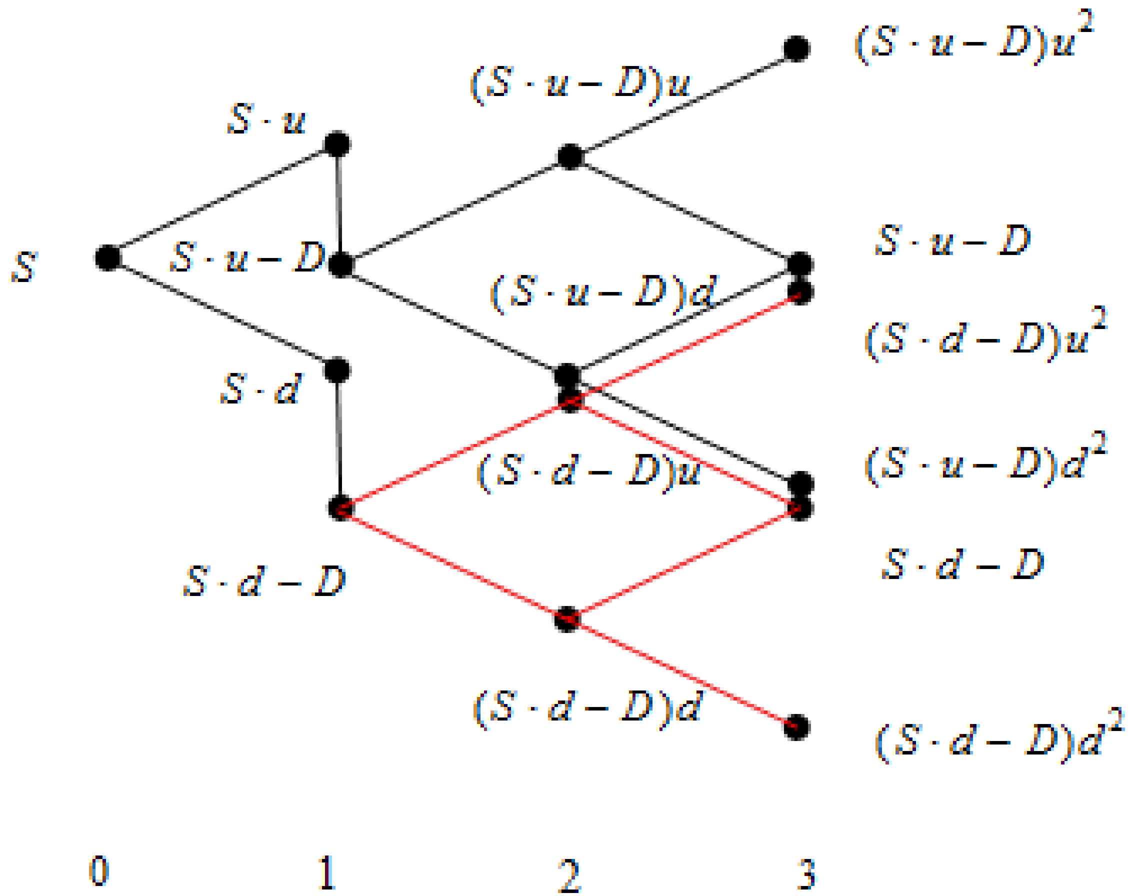
$$C = e^{-r \cdot \Delta t} \left[\frac{e^{(r - \delta) \cdot \Delta t} - d}{u - d} C_u + \frac{u - e^{(r - \delta) \cdot \Delta t}}{u - d} C_d \right]$$

$$\boxed{C = e^{-r \cdot \Delta t} \left(q \cdot C_u + (1 - q) \cdot C_d \right)}$$

$$q = \frac{e^{(r - \delta) \cdot \Delta t} - d}{u - d}$$

Incoherent binomial option tree

(the underlying asset pays predictable income**)**



Example

- Find the fair value of an European call option using the incoherent binomial option tree if the underlying asset pays dividend of 2 PLN in half a month. The initial stock price is 50 PLN, the strike price of 48 PLN is expiring at the end of the third month, the continuously compounded risk-free interest rate is 10% per annum, and the stock volatility is 20%

$$u = e^{0.2\cdot\sqrt{1/12}} = 1.059434$$

$$q = 0.558$$

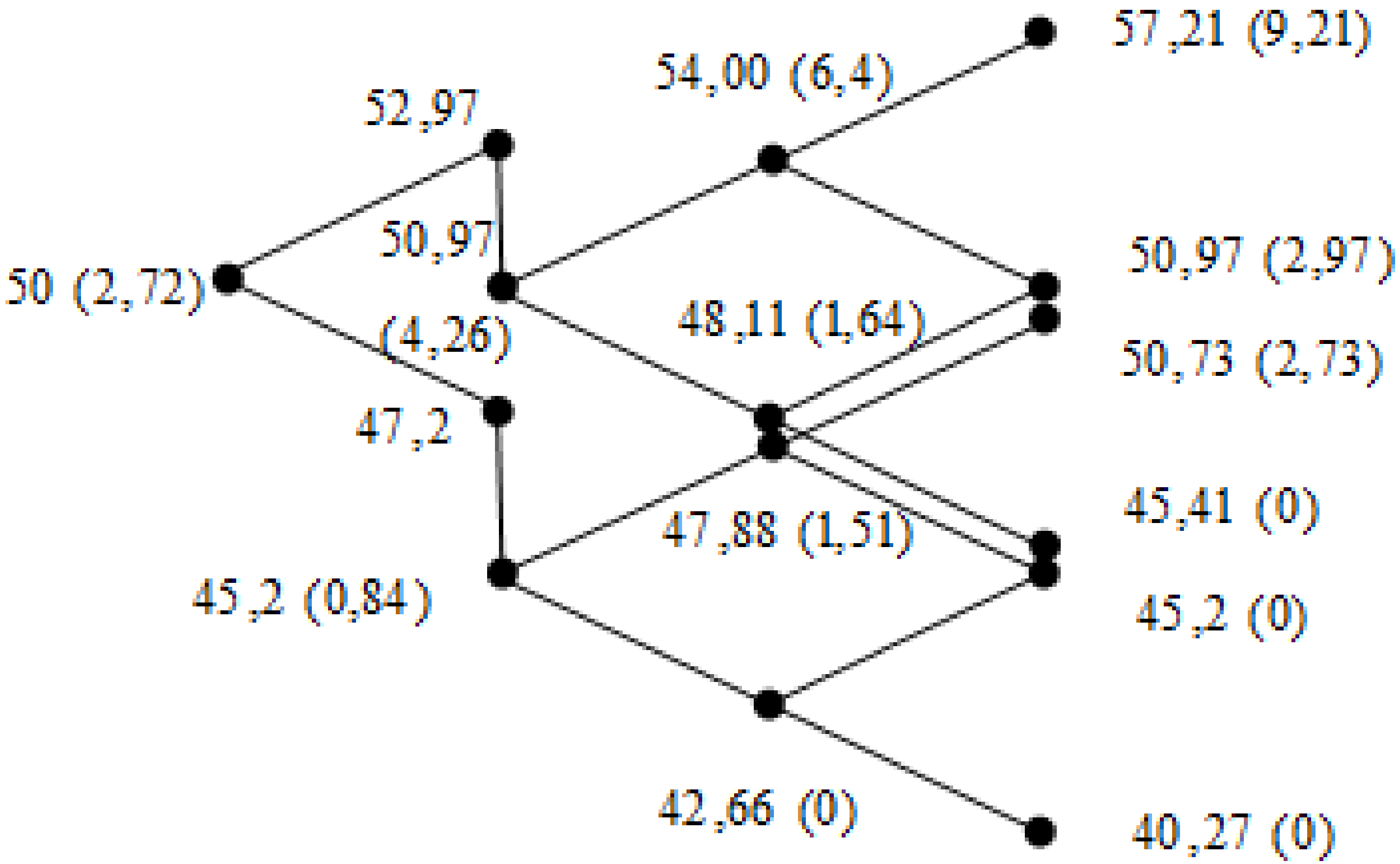
$$d = 1/u = 0.9439$$

$$1 - q = 0.442$$

$$e^{r\cdot\Delta t} = e^{0.1/12} = 1.008368$$

$$C = e^{-r\cdot\Delta t}\left(q\cdot C_u + (1-q)\cdot C_d\right)$$

$$q = \frac{e^{r\cdot\Delta t} - d}{u - d}$$


57,21 (9,21)
54,00 (6,4)
52,97
50,97
(4,26)
50 (2,72)
48,11 (1,64)
50,97 (2,97)
50,73 (2,73)
47,2
47,88 (1,51)
45,41 (0)
45,2 (0,84)
45,2 (0)
42,66 (0)
40,27 (0)

$D = 2, \quad K = 48$

Example - coherent binomial option tree (American put option)

$$S = 52 \quad K = 50 \quad D = 2.06 \quad r = 0.1 \quad \sigma = 0.4$$

$$\tau = 3.5 \qquad T = 5$$

$$u = e^{0.4 \cdot \sqrt{1/12}} = 1.122401$$

$$q = 0.507319$$

$$d = 1/u = 0.890947$$

$$1 - q = 0.492681$$

$$e^{r \cdot \Delta t} = e^{0.1/12} = 1.008368$$

$$P = e^{-r \cdot \Delta t}\left(q \cdot P_u + (1-q) \cdot P_d\right)$$

$$q = \frac{e^{r \cdot \Delta t} - d}{u - d}$$

S^*

$$D \cdot e^{-t \cdot \Delta t \cdot r} = 2.06 \cdot e^{-3.5 \cdot \frac{1}{12} \cdot 0.1} = 2.00078$$

49.99922	56.12	62.99	70.7	79.35	89.06
	44.55	49.999	56.12	62.99	70.7
		39.69	44.55	49.999	56.12
			35.36	39.69	44.55
				31.5	35.36
					28.07
$+2.00078$	$+2.017527$	$+2.024938$	$+2.051435$		
$+De^{-3.5 \cdot \frac{1}{12} \cdot 0.1}$	$+De^{-2.5 \cdot \frac{1}{12} \cdot 0.1}$	$+De^{-1.5 \cdot \frac{1}{12} \cdot 0.1}$	$+De^{-0.5 \cdot \frac{1}{12} \cdot 0.1}$		

S

52	58.14	65.01	72.75	79.35	89.06
	46.56	52.02	58.17	62.99	70.7
		41.71	46.598	49.999	56.12
			37.41	39.69	44.55
				31.5	35.36
					28.07

American put option

4.44	2.16	0.64	0	0	0
	6.86	3.77	1.3	0	0
		10.16	6.38	2.66	0
			14.22	10.31	5.45
				18.5	14.64
					21.93

Formula	K – S
0	– 29.35
0	–12.988
2.66	0
9.896	10.31
18.08	18.496