Mathematical Economics MME2/2 – 2018/2019 (sample of the final exam)

1. A consumer has a utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$ and the budget constraint is $p_1 x_1 + p_2 x_2 = I$, $p_1, p_2, I > 0$. What are the Marshallian demand functions?

(1 point)

2. Solve the expenditure minimization problem

$$\min_{x_1, x_2} \quad p_1 x_1 + p_2 x_2 \qquad\qquad p_1, p_2, u > 0.$$
$$u = x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$$

(1 point)

3. For the technologies $y = \min\left\{\frac{1}{4} x_1, 2x_2\right\}$ and $y = \frac{x_1}{2} + x_2$ compute:

a) the total cost and the cost of production of 1 unit of output,

b) the marginal and average cost.

Assume that prices of inputs are (4, 6).

(1 point)

4. Suppose that we have two firms that face linear demand curve $p = 200 - \frac{1}{2}(y_1 + y_2)$ and their cost functions are $c_1(y_1) = \frac{1}{2} y_1^2$, $c_2(y_2) = 10 y_2$, respectively.

a) Compute the Cournot equilibrium amount of output for each firm and firms' profits.

b) If firm 2 behaves as a follower and firm 1 behaves as a leader, compute the Stackelberg equilibrium amount of output for each firm and firms' profits.

(1 point)

5. The traders' utilities are given by $u^1(x_1, x_2) = x_1 x_2^2$ and $u^2(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Their initial endowments are the following $a^1 = (2,2)$ and $a^2 = (4,4)$. Traders come to a market and exchange commodities to maximize their utilities. Compute the price vector in equilibrium. Compare the utilities before and after the exchange.

(1 point)