Modern Growth Theories
Lecture 3

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Harrod model

- Dynamic extension of the Keynesian analysis of static equilibrium (short-run macroeconomics).
- Harrod and Domar independently developed theories that relate an economy's rate of growth to its capital stock.
- Keynes emphasized the impact of investment on aggregate demand.
- Harrod and Domar emphasized how investment spending also increased an economy's productive capacity.

Harrod model

- Roy F. Harrod (1900-1978)
- *An Essay in Dynamic Theory*, „Economic Journal”, 1939, 49(193), 14-33.

AN ESSAY IN DYNAMIC THEORY

1. THE following pages constitute a tentative and preliminary attempt to give the outline of a "dynamic" theory. Static theory consists of a classification of terms with a view to systematic thinking, together with the extraction of such knowledge about the adjustments due to a change of circumstances as is yielded by the "laws of supply and demand." It has for some time appeared to me that it ought to be possible to develop a similar classification and system of axioms to meet the situation in which certain forces are operating steadily to increase or decrease certain magnitudes in the system. The consequent "theory" would not profess to determine the course of events in detail, but should provide a framework of concepts relevant to the study of change analogous to that provided by static theory for the study of rest.

Harrod model – assumptions

- Closed a two-sector economy (households and firms).
- There is the absence of government interference.
- There is a fixed proportion of capital and labour in the productive process (constant capital-labour ratio).
- The capital coefficient – the ratio of capital stock to income is assumed to be fixed (constant capital-output ratio).
- The marginal propensity to save remains constant.
- The average propensity to save is equal to marginal propensity to save.
- There are no lags in adjustments between investments and creation of productive capacity.

Harrod model – assumptions

- An economy with single commodity Y
- A Leontief technology

$$Y = F(K, L) = \min\left\{\frac{K}{v}, \frac{L}{a}\right\}$$

- To produce one unit of goods we need v units of capital K and a units of labour L, where v and a are numbers.
- Constant returns to scale
- K/Y and L/Y are constant

Harrod model – assumptions

- If the available capital stock and labour force happen to be such that

$$\frac{K}{v} = \frac{L}{a}$$

than all workers and machines are fully employed.

Harrod model – assumptions

- If K and L are such that $\frac{K}{v} > \frac{L}{a}$

then only quantity of capital $\frac{v}{a}L$
is used, and the remainder remains idle.

- Conversely, if $\frac{K}{v} < \frac{L}{a}$

then only the amount of labour $\frac{a}{v}K$
is used and the remainder is unemployed

Harrod model – assumptions

- A fixed-proportions production function, fixed coefficient production function.
- *K/L* is constant if production is efficient.
- In most cases, we will have either excess capital or excess labour. There is plenty of labour, capital is the limiting factor (capital is binding constraint).

$$Y = \frac{K}{v}$$

$$\frac{K}{Y} = v$$

$$\frac{Y}{K} = \frac{1}{v}$$

$$\frac{\Delta Y}{\Delta K} = \frac{1}{v}$$

v – capital-output ratio is constant

Harrod model – assumptions

- A closed economy (no trade or capital flows)
- Savings *S* must be used for investment *I*

$$S_t = I_t$$

Plans to invest equal plans to save – the condition for income and output to be in equilibrium

- A constant fraction of income is saved – *s* the savings rate

$$S_t = s \cdot Y_t$$

Harrod model

- What must be the rate of growth of income for plans to invest to equal plans to save in order to ensure moving equilibrium in a growing economy through time?

- The model describes the mobilization of savings for generating sufficient investment to accelerate economic growth.

Harrod model

- Changes in capital stock comes from investment and the depreciation of the capital stock. Net investment is defined as the change in the capital stock.

$$K_{t+1} = K_t + I_t - \delta \cdot K_t$$

$$\Delta K = K_{t+1} - K_t \qquad \Delta K = I_t - \delta \cdot K_t$$

$$g_K = \frac{K_{t+1} - K_t}{K_t}$$

Harrod model

$$g_K = \frac{K_{t+1} - K_t}{K_t} = \frac{I_t - \delta \cdot K_t}{K_t} = \frac{I_t}{K_t} - \delta$$

$$g_K = \frac{S_t}{K_t} - \delta = \frac{s \cdot Y_t}{K_t} - \delta = \frac{s}{v} - \delta$$

$$\boxed{g_K = \frac{s}{v} - \delta}$$

Harrod model

$$\frac{K_t}{Y_t} = \nu \qquad \frac{\Delta Y}{\Delta K} = \frac{1}{\nu}$$

$$\frac{1}{\nu} = \frac{Y_{t+1} - Y_t}{K_{t+1} - K_t} = \frac{Y_{t+1} - Y_t}{K_t + I_t - \delta \cdot K_t - K_t}$$

$$\frac{1}{\nu} = \frac{Y_{t+1} - Y_t}{s \cdot Y_t - \delta \cdot K_t} = \frac{\Delta Y}{s \cdot Y_t - \delta \cdot \nu \cdot Y_t}$$

$$\frac{1}{\nu} = \frac{Y_{t+1} - Y_t}{s \cdot Y_t - \delta \cdot K_t} = \frac{\Delta Y}{(s - \delta \cdot \nu) Y_t}$$

$$g_Y = \frac{Y_{t+1} - Y_t}{Y_t} = \frac{s - \delta \cdot \nu}{\nu} = \boxed{\frac{s}{\nu} - \delta}$$

Harrod model

$$g_Y = \frac{s}{v} - \delta$$

$\frac{dg_Y}{ds} > 0$ Savings increase growth

$\frac{dg_Y}{dv} < 0$ Efficiency increases growth

$\frac{dg_Y}{d\delta} < 0$ Depreciation decreases growth

Harrod model

$$g_Y = \frac{s}{v}$$

$s = 6\%$

$v = 3$

$$g_Y = \frac{6\%}{3} = 2\%$$

$s = 15\%$

$v = 3$

$$g_Y = \frac{15\%}{3} = 5\%$$

$$\frac{\Delta K}{\Delta Y} = \nu$$

Harrod model

- Capital-output ratio – efficiency with which capital is used, a measure of capital efficiency, a high *v* implies a high increase in capital stock relative to the increase in GDP.
- ICOR – incremental capital-output ratio (the ratio of investments to growth which is equal to 1 divided by the marginal product of capital).
- The higher ICOR, the lower the productivity of capital

$$ICOR = \frac{I}{\Delta Y}$$

Harrod model – assumptions

- Changes in income induce investment
- *ICOR* is a metric that assesses the marginal amount of investment capital necessary for an entity to generate the next unit of production.
- It relates new investment to the change aggregate demand.

$$I_t = ICOR \cdot \left(Y_t - Y_{t-1} \right)$$

Harrod model

$$S_t = I_t$$

The economy is in equilibrium when desired investment equals actual savings

$$S_t = s \cdot Y_{t-1}$$

$$I_t = ICOR \cdot \left(Y_t - Y_{t-1}\right)$$

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{s}{ICOR}$$

Harrod model

$$Y_t = \left(1 + \frac{s}{ICOR}\right) \cdot Y_{t-1}$$

$$Y_t = \left(1 + \frac{s}{ICOR}\right)^t \cdot Y_0$$

$$g_Y = \frac{s}{ICOR} \qquad g_Y = \frac{s}{ICOR} - \delta$$

- **The growth rate of GDP is directly (positively) related to the saving ratio**, i.e., the more an economy is able to save – and therefore invest – out of a given GDP, the greater will be the growth of that GDP.

- **The growth rate of GDP is indirectly (negatively) related to the economy's capital-output ratio**, i.e., the higher is ICOR, the lower will be the rate of GDP growth.

Harrod model

- Capital accumulation and savings were the two main ingredients necessary for an economy to grow.

- Economic growth can be accelerated by
  - changing the saving rate
  - improving technology.

Harrod model

- **Actual growth rate** – an economy has some growth rate at which it is actually growing. It may not be steady growth.

- **Warranted (desired) growth rate** is the growth rate at which all savings are absorbed into investment. *Ex ante S* should be exactly equal to the *ex post I*.

- **Natural growth rate** is the rate of economic growth required to maintain full employment.

Harrod model

$$g_Y = \frac{s}{ICOR}$$

- Knife-edge dynamics (boom, inflationary situation)

$$g_a > g_w \quad \Rightarrow \quad \frac{s}{ICOR_{actual}} > \frac{s}{ICOR_{desired}}$$

$$g_a > g_w \quad \Rightarrow \quad ICOR_{desired} > ICOR_{actual}$$

- Investors have to increase *ICOR*, they increase investment, but it leads to g_a increase, gap between actual and warranted growth rates increases.

Harrod model

$$g_Y = \frac{s}{ICOR}$$

- Knife-edge dynamics (recession, unemployment)

$$g_a < g_w \quad \Rightarrow \quad \frac{s}{ICOR_{actual}} < \frac{s}{ICOR_{desired}}$$

$$g_a < g_w \quad \Rightarrow \quad ICOR_{desired} < ICOR_{actual}$$

- Investors think that they have overinvested, and reduce their investment, actual growth rate falls

Harrod model

- When investors/producers think they are producing more they are actually producing less.

$$g_a < g_w$$

- When investors/producers think they are producing less they are actually producing more.

$$g_a > g_w$$

Harrod model

- Harrod concludes that because of wrong reasoning of producers, it is not possible to achieve steady growth.
- Growth in capitalist economy is basically unstable.

Harrod model

- Natural rate of growth – the maximum rate of growth of the economy (long term growth rate of population + neutral technical progress)

$$g_n = g_L + g_T$$

- Full employment $g_a = g_w = g_n$
- Never achieved

Harrod model

- Knife-edge dynamics
- If growth rate of labour force > warranted growth rate – then chronic unemployment
- If growth rate of labour force < warranted growth rate – then chronic labour shortage

Harrod model – weaknesses

- Savings as sufficient (investment is uncertain)
- Rigid assumption of fixed proportions (zero substitutability between capital and labour)
- No diminishing returns, no factor substitution
- No technological change
- Unrealistic lack of response of ICOR to policy (development raises ICOR)

Harrod model – weaknesses

- Difficult to increase the savings ratio in lower-income countries. Since many developing countries have low marginal propensities to save, countries suffer from a persistent **domestic savings gap.**
- Many developing countries also **lack a sound financial system**. Increased saving by households does not necessarily mean there will be invested.
- Efficiency gains that reduce the capital/output ratio are difficult to achieve in developing countries due to **weaknesses in human capital**, causing capital to be used inefficiently.

Harrod model – weaknesses

- Research and development (R&D) are low in developing countries to improve the capital/output ratio.

- Borrowing from overseas to fill the savings gap causes external debt repayment problems later.

Harrod model

- The simple model was convenient for economic planners seeking a specific target growth rate; the formula could be used to justify the foreign aid and government taxation if private domestic saving was not sufficient.

Harrod model

- Used to calculate financing gaps – how much foreign assistance to achieve a particular rate of output growth?

$$g_Y = \frac{s}{v} - \delta$$

$$g_Y = \frac{s_{private} + s_{public} + s_{foreign}}{v} - \delta$$

The Domar model

- Evsey D. Domar (1914-1997)

- Evsey D. Domar (1946), Capital Expansion, Rate of Growth, and Employment, *Econometrica* 14(2), 137-147.

The Domar model

- Investment plays a key role in the process of economic growth.

- Investment creates income (a demand effect) and raises the productive capacity of an economy by increasing its capital stock.

The Domar model

- Investment changes the economy's supply side as well as the demand side, and full employment could be maintained only if investment and the other sources of aggregate demand grew just fast enough to exactly absorb the increased output that the new investment made possible.

- At what rate investment should increase in order to make the increase in income equal to the increase in productive capacity, so that full employment is maintained?

The Domar model – the demand side

- The demand effect of a change in *I(t)* operates through the multiplier process. An increase in *I(t)* will raise the rate of income flow *Y(t)* by a multiple in of the increment in *I(t)*.

$$dY = \frac{1}{MPS} \cdot dI \qquad \frac{dY}{dI} = \frac{1}{MPS} = m_I$$

$$\frac{dY}{dt} = \frac{1}{MPS} \cdot \frac{dI}{dt} = m_I \cdot \frac{dI}{dt}$$

I(t) is the only expenditure flow that influences the rate of income flow

The Domar model – the supply side

- The capacity effect of investment is to be measured by the change in the rate of potential output the economy is capable of producing.

Y_p – potential output (maximum output associated with given stock of capital)

- Assuming a constant output-capital ratio (capital coefficient)

$$\frac{Y_p}{K} = \rho \qquad\qquad Y_p = \rho \cdot K$$

The Domar model – the supply side

- With a capital stock *K(t)* the economy is potentially capable of producing an annual product, or income, amounting to $Y_p = \rho \cdot K$

$$\frac{dY_p}{dt} = \rho \cdot \frac{dK}{dt} = \rho \cdot I$$

If capital does not depreciate, then the change in the capital stock is exactly equal to investment.

The Domar model

- Equilibrium – productive capacity is fully utilized. The aggregate demand is equal to the potential output producible in a year

$$Y = Y_p$$

$$\frac{dY}{dt} = m_I \cdot \frac{dI}{dt}$$

$$\frac{dY_p}{dt} = \rho \cdot I$$

$$\frac{dY}{dt} = \frac{dY_p}{dt}$$

$$\frac{dI}{dt} = \frac{\rho}{m_I} \cdot I = MPS \cdot \rho \cdot I$$

$$I(t) = I(0)e^{MPS \cdot \rho \cdot t}$$

The Domar model

- In order to maintain the balance between capacity and demand over time, the rate of investment flow must grow precisely at the exponential rate of $MPS \cdot \rho$

- What will happen if the actual rate of growth of investment *r* differs from the required rate

$MPS \cdot \rho$

The Domar model – the razor's edge

- Domar defined a coefficient of utilization

$$u = \lim_{t \to \infty} \frac{Y(t)}{Y_p(t)} = \frac{dY/dt}{dY_p/dt} = \frac{r}{MPS \cdot \rho}$$

u=1 means full utilization of capacity $r = MPS \cdot \rho$

$$I(t) = I(0)e^{r \cdot t}$$

$$\frac{dY}{dt} = m_I \cdot \frac{dI}{dt} = \frac{1}{MPS} \cdot r \cdot I(0)e^{r \cdot t}$$

$$\frac{dY_p}{dt} = \rho \cdot I = \rho \cdot I(0)e^{r \cdot t}$$

The Domar model – the razor's edge

$$r = MPS \cdot \rho$$

Full utilization of the capacity equating the actual growth rate to required growth rate

$$r > MPS \cdot \rho \quad \Rightarrow \frac{dY}{dt} > \frac{dY_p}{dt}$$

Aggregate demand surpasses the productive capacity; this implies the shortage of capacity which leads to inflationary pressure in economy; investors will invest more which means increase in *r*

$$r < MPS \cdot \rho \quad \Rightarrow \frac{dY}{dt} < \frac{dY_p}{dt}$$

If the actual growth of investment lags behind the required rate, we will encounter a capacity surplus leading to deflationary situations; investors will reduce the investment which means decrease in *r*