Modern Growth Theories
Lecture 5

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The Domar model

- Evsey D. Domar (1914-1997)

- Evsey D. Domar (1946), Capital Expansion, Rate of Growth, and Employment, *Econometrica* 14(2), 137-147.

The Domar model

- Investment plays a key role in the process of economic growth.

- Investment creates income (a demand effect) and raises the productive capacity of an economy by increasing its capital stock.

The Domar model

- Investment changes the economy’s supply side as well as the demand side, and full employment could be maintained only if investment and the other sources of aggregate demand grew just fast enough to exactly absorb the increased output that the new investment made possible.

- At what rate investment should increase in order to make the increase in income equal to the increase in productive capacity, so that full employment is maintained?

The Domar model – the demand side

- The demand effect of a change in *I(t)* operates through the multiplier process. An increase in *I(t)* will raise the rate of income flow *Y(t)* by a multiple in of the increment in *I(t)*.

$$dY = \frac{1}{MPS} \cdot dI \qquad \frac{dY}{dI} = \frac{1}{MPS} = m_I$$

$$\frac{dY}{dt} = \frac{1}{MPS} \cdot \frac{dI}{dt} = m_I \cdot \frac{dI}{dt}$$

I(t) is the only expenditure flow that influences the rate of income flow

The Domar model – the supply side

- The capacity effect of investment is to be measured by the change in the rate of potential output the economy is capable of producing.

Y_p – potential output (maximum output associated with given stock of capital)

- Assuming a constant output-capital ratio (capital coefficient)

$$\frac{Y_p}{K} = \rho \qquad\qquad Y_p = \rho \cdot K$$

The Domar model – the supply side

- With a capital stock *K(t)* the economy is potentially capable of producing an annual product, or income, amounting to $Y_p = \rho \cdot K$

$$\frac{dY_p}{dt} = \rho \cdot \frac{dK}{dt} = \rho \cdot I$$

If capital does not depreciate, then the change in the capital stock is exactly equal to investment.

The Domar model

- Equilibrium – productive capacity is fully utilized. The aggregate demand is equal to the potential output producible in a year

$$Y = Y_p$$

$$\frac{dY}{dt} = m_I \cdot \frac{dI}{dt}$$

$$\frac{dY}{dt} = \frac{dY_p}{dt}$$

$$\frac{dY_p}{dt} = \rho \cdot I$$

$$\frac{dI}{dt} = \frac{\rho}{m_I} \cdot I = MPS \cdot \rho \cdot I$$

$$I(t) = I(0)e^{MPS \cdot \rho \cdot t}$$

The Domar model

- In order to maintain the balance between capacity and demand over time, the rate of investment flow must grow precisely at the exponential rate of $MPS \cdot \rho$

- What will happen if the actual rate of growth of investment *r* differs from the required rate

  $MPS \cdot \rho$

The Domar model – the razor's edge

- Domar defined a coefficient of utilization

$$u = \lim_{t\to\infty} \frac{Y(t)}{Y_p(t)} = \frac{dY/dt}{dY_p/dt} = \frac{r}{MPS\cdot\rho}$$

u=1 means full utilization of capacity $r = MPS\cdot\rho$

$$\frac{dY}{dt} = m_I \cdot \frac{dI}{dt} = \frac{1}{MPS}\cdot r\cdot I(0)e^{r\cdot t}$$

$$I(t) = I(0)e^{r\cdot t}$$

$$\frac{dY_p}{dt} = \rho\cdot I = \rho\cdot I(0)e^{r\cdot t}$$

The Domar model – the razor's edge

$$r = MPS \cdot \rho$$

Full utilization of the capacity equating the actual growth rate to required growth rate

$$r > MPS \cdot \rho \quad \Rightarrow \frac{dY}{dt} > \frac{dY_p}{dt}$$

Aggregate demand surpasses the productive capacity; this implies the shortage of capacity which leads to inflationary pressure in economy; investors will invest more which means increase in r

$$r < MPS \cdot \rho \quad \Rightarrow \frac{dY}{dt} < \frac{dY_p}{dt}$$

If the actual growth of investment lags behind the required rate, we will encounter a capacity surplus leading to deflationary situations; investors will reduce the investment which means decrease in r

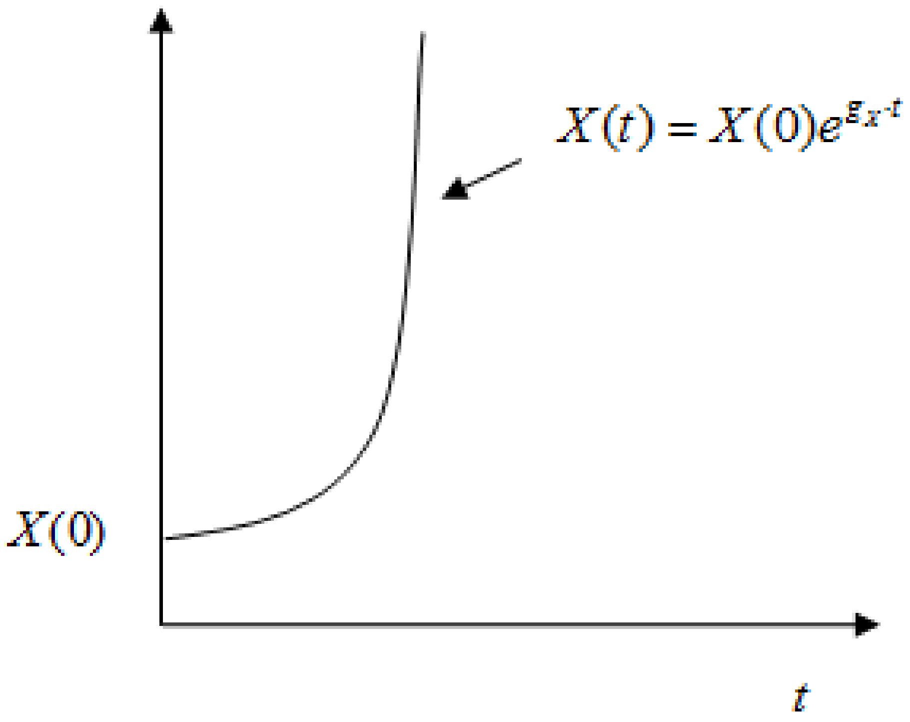
The Solow growth model

- Robert Solow (1924-)
- Nobel Memorial Prize in Economic Sciences, 1987

- Solow R. (1956), A Contribution to the Theory of Economic Growth, *Quarterly Journal of Economics*, 70(1), 65-94.

Growth rate

$$g_X = \frac{\dot{X}(t)}{X(t)} \equiv \frac{d \ln X(t)}{dt} \quad \text{where} \quad \dot{X}(t) \equiv \frac{dX(t)}{dt}$$



Properties of growth rate

Let $X(t)$, $Y(t)$, then

1. $g_{X\cdot Y} = g_X + g_Y$,
2. $g_{X/Y} = g_X - g_Y$,
3. $g_{X+Y} = \dfrac{X}{X+Y} g_X + \dfrac{Y}{X+Y} g_Y$,
4. $g_{X-Y} = \dfrac{X}{X-Y} g_X - \dfrac{Y}{X-Y} g_Y$,

where g_X is the growth rate of $X(t)$,

g_Y is the growth rate of $Y(t)$.

The Solow growth model

$$Y(t) = F(K(t), A(t), L(t)),$$

$Y(t)$ corresponds to GDP,
$K(t)$ is a capital, $L(t)$ is a labour force,
$A(t)$ means technical change.

$Y(t) = F(K(t), A(t)L(t))$ - labour augmenting technology progress (Harrod neutral).

$Y(t) = F(A(t)K(t), L(t))$ - technical change is Solow neutral (capital augmenting technical change)

$Y(t) = A(t)F(K(t), L(t))$ - technical change is Hicks neutral

Properties of neoclassical production function

1. $F: \Re_+^2 \to \Re_+$ is twice differentiable on $\text{int}\,\Re_+^2$.

2. $F(K,0) = F(0, AL) = 0$, $\forall\ K, AL \in \Re_+$. $F(0,0) = 0$

3. $\forall \lambda > 0$, $\forall (K, AL) \in \Re_+^2$ $F(\lambda K, \lambda AL) = \lambda F(K, AL)$ - constant returns to scale.

4. $\forall\ (K, AL), (K', AL') \in \Re_+^2$ $(K, AL) > (K', AL') \Rightarrow F(K, AL) > F(K', AL')$

   - an increasing function

5. A concave function: $\forall \lambda \in (0,1)$, $\forall\ (K, AL), (K', AL') \in \Re_+^2$

$$F(\lambda K + (1-\lambda)K', \lambda AL + (1-\lambda)AL') \geq \lambda F(K, AL) + (1-\lambda)F(K', AL')$$

6. Inada conditions: $\lim\limits_{K \to 0} \dfrac{\partial F(K, AL)}{\partial K} = \lim\limits_{(AL) \to 0} \dfrac{\partial F(K, AL)}{\partial (AL)} = +\infty$

$$\lim_{K \to +\infty} \frac{\partial F(K, AL)}{\partial K} = \lim_{(AL) \to +\infty} \frac{\partial F(K, AL)}{\partial (AL)} = 0 .$$

The Solow model with labour-augmenting technological progress

- Production function is: $Y = K^{\alpha}(AL)^{1-\alpha}$

  $\alpha \in (0, 1)$ is capital share of income. Constant returns to scale.

- Aggregate capital accumulation: $\dot{K} = sY - \delta K$,

  where s is an exogenous saving rate (investment rate).

  δ is a depreciation rate.

- $\frac{\dot{A}}{A} = g$ is an exogenous rate of technology progress ($A(t) = A(0)e^{gt}$).

  Efficiency of labour grows at a constant rate.

- Labour force grows at a constant rate $\frac{\dot{L}}{L} = n$

  (n is a constant growth rate of labour force). $L(t) = L(0)e^{nt}$

If $y = \dfrac{Y}{A \cdot L}$ is product per efficient unit of labour,

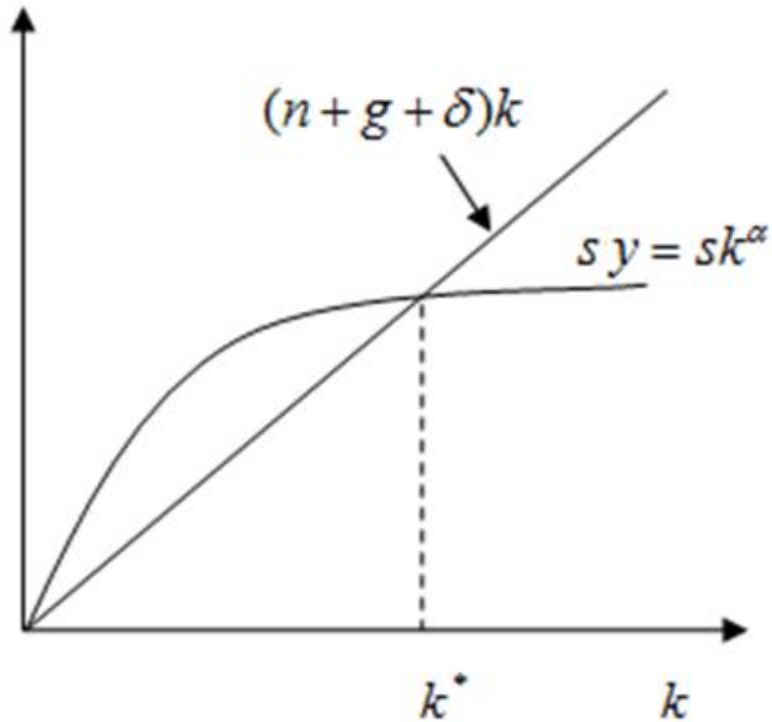
$k = \dfrac{K}{A \cdot L}$ is capital per efficient unit of labour then

$$y = k^{\alpha}$$

$$\dot{k} = sy - (n + g + \delta)k$$

Steady state equilibrium

The steady state is at point where $\dot{k} = 0$.



$$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

In the steady state growth rates are the following:

$g_K = n + g$, $g_Y = n + g$, $g_{K/L} = g$, $g_{Y/L} = g$

An economy approaches to steady state

$$k(t) \to \left(\frac{s}{n+g+\delta} \right)^{\frac{1}{1-\alpha}} = k^*$$

Note:

Solving the equation $\frac{dk}{dt} = sk^{\alpha} - (n+g+\delta)k$ we obtain

$$k(t)^{(1-\alpha)} = \frac{s}{n+g+\delta} + \left(k(0)^{(1-\alpha)} - \frac{s}{n+g+\delta} \right) e^{-(1-\alpha)(n+g+\delta)t}$$

The rate of convergence is $\lambda = (1-\alpha)(n+g+\delta)$

Note: $\dot{k} = \left.\frac{d\dot{k}}{dk}\right|_{k=k^*} (k-k^*) = -(1-\alpha)(n+g+\delta)(k-k^*)$,

$k(t) - k^* = (k(0) - k^*)e^{-\lambda \cdot t}$, where $\lambda = (1-\alpha)(n+g+\delta)$.