

## Mathematical Economics (2017/2018)

### Exercises 1

1. Draw the indifference curve:

a)  $u(x_1, x_2) = \frac{1}{2} \min\{x_1, 2x_2\}$ ,  $u(x_1, x_2) = 4$ ;

b)  $u(x_1, x_2) = \min\{3x_1 + x_2, x_1 + 3x_2\}$  passing through point (4, 4);

c)  $u(x_1, x_2) = \min\{x_1 + 7x_2, 4x_1 + x_2\}$ ,  $u(x_1, x_2) = 9$ .

2. Solve the following utility maximization problem

a) 
$$\max_{x_1, x_2} \left( \frac{1}{2}x_1 + 1 \right) (x_2 + 2)$$
  

$$2x_1 + x_2 = 8$$

b) 
$$\max_{x_1, x_2} x_1 + x_1x_2 + x_2 + 1$$
  

$$x_1 + 3x_2 = 9$$

c) 
$$\max_{x_1, x_2} x_1^{0.6} x_2^{0.4}$$
  

$$3x_1 + 4x_2 = 5$$

d) 
$$\max_{x_1, x_2} 3(x_1 + 1)(x_2 + 2)$$
  

$$p_1x_1 + p_2x_2 = I \quad p_1, p_2, I > 0.$$

3. Find the demanded bundle for a consumer whose utility function and budget constraint are the following

| Utility function   | Budget constraint           |
|--|-----------------------------|
| a) $u(x_1, x_2) = (3x_1 + 3)(x_2 + 3)$                         | $x_1 + 2x_2 = 6$            |
| b) $u(x_1, x_2) = \left( \frac{1}{2}x_1 + 2 \right) (x_2 + 4)$ | $2x_1 + x_2 = 10$           |
| c) $u(x_1, x_2) = \left( \frac{1}{3}x_1 + 3 \right) (x_2 + 3)$ | $3x_1 + x_2 = 30$           |
| d) $u(x_1, x_2) = (x_1 + 2) \left( \frac{1}{4}x_2 + 2 \right)$ | $x_1 + 2x_2 = 20$           |
| e) $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{4}}$         | $\frac{1}{3}x_1 + 5x_2 = 3$ |
| f) $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2$                       | $x_1 + 2x_2 = 6$            |

4. Find the demand functions  $x_1(p_1, p_2, I)$  and  $x_2(p_2, p_1, I)$  if the budget constraint is  $p_1x_1 + p_2x_2 = I$ ,  $p_1, p_2, I > 0$  and the utility function is given by

a)  $u(x_1, x_2) = \frac{1}{3}(x_1 + 2)(x_2 + 1),$

b)  $u(x_1, x_2) = 3x_1^{\frac{1}{4}}x_2^{\frac{1}{2}},$

c)  $u(x_1, x_2) = x_1x_2^3,$

d)  $u(x_1, x_2) = a \ln x_1 + (1-a) \ln x_2, a \in (0,1),$

e)  $u(x_1, x_2) = x_1^2 + x_2^2,$

f)  $u(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)^3,$

g)  $u(x_1, x_2) = 2 \min\{4x_1 + x_2, x_1 + 4x_2\},$

h)  $u(x_1, x_2) = \min\{4x_1 + x_2, x_1 + 7x_2\}.$

5. Solve the expenditure minimization problem

a) 
$$\min_{x_1, x_2} p_1x_1 + p_2x_2$$

$$u = x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}$$

b) 
$$\min_{x_1, x_2} p_1x_1 + p_2x_2$$

$$u = x_1^{\frac{1}{4}}x_2^{\frac{1}{2}}$$

$p_1, p_2, u > 0.$