Mathematical Economics (2018/2019)

Exercises 1

1. Draw the indifference curve:

a) $u(x_1, x_2) = \frac{1}{2}\min\{x_1, 2x_2\}, \quad u(x_1, x_2) = 4;$

b) $u(x_1, x_2) = \min\{3x_1 + x_2,\ x_1 + 3x_2\}$ passing through point $(4, 4)$;

c) $u(x_1, x_2) = \min\{x_1 + 7x_2, 4x_1 + x_2\}, \quad u(x_1, x_2) = 9.$

2. Solve the following utility maximization problem

a) $\max_{x_1, x_2} \left(\frac{1}{2}x_1 + 1\right)(x_2 + 2)$
$2x_1 + x_2 = 8$

b) $\max_{x_1, x_2} x_1 + x_1x_2 + x_2 + 1$
$x_1 + 3x_2 = 9$

c) $\max_{x_1, x_2} x_1^{0,6}x_2^{0,4}$
$3x_1 + 4x_2 = 5$

d) $\max_{x_1, x_2} 3(x_1 + 1)(x_2 + 2)$
$p_1x_1 + p_2x_2 = I$
$p_1, p_2, I > 0.$

3. Find the demanded bundle for a consumer whose utility function and budget constraint are the following

Utility function	Budget constraint
a) $u(x_1, x_2) = (3x_1 + 3)(x_2 + 3)$	$x_1 + 2x_2 = 6$
b) $u(x_1, x_2) = \left(\frac{1}{2}x_1 + 2\right)(x_2 + 4)$	$2x_1 + x_2 = 10$
c) $u(x_1, x_2) = \left(\frac{1}{3}x_1 + 3\right)(x_2 + 3)$	$3x_1 + x_2 = 30$
d) $u(x_1, x_2) = (x_1 + 2)\left(\frac{1}{4}x_2 + 2\right)$	$x_1 + 2x_2 = 20$
e) $u(x_1, x_2) = x_1^{\frac{1}{2}}x_2^{\frac{1}{4}}$	$\frac{1}{3}x_1 + 5x_2 = 3$
f) $u(x_1, x_2) = x_1^{\frac{1}{2}}x_2$	$x_1 + 2x_2 = 6$

4. Find the demand functions $x_1(p_1, p_2, I)$ and $x_2(p_2, p_1, I)$ if the budget constraint is $p_1x_1 + p_2x_2 = I$, $p_1, p_2, I > 0$ and the utility function is given by

a) $u(x_1, x_2) = \frac{1}{3}(x_1 + 2)(x_2 + 1)$,

b) $u(x_1, x_2) = 3x_1^{\frac{1}{4}}x_2^{\frac{1}{2}}$,

c) $u(x_1, x_2) = x_1 x_2^3$,

d) $u(x_1, x_2) = a\ln x_1 + (1-a)\ln x_2$, $a \in (0,1)$,

e) $u(x_1, x_2) = x_1^2 + x_2^2$,

f) $u(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}} \right)^3$,

g) $u(x_1, x_2) = 2\min\{4x_1 + x_2, x_1 + 4x_2\}$,

h) $u(x_1, x_2) = \min\{4x_1 + x_2, x_1 + 7x_2\}$.

5. Solve the expenditure minimization problem

a) $\min_{x_1, x_2} \; p_1x_1 + p_2x_2$ subject to $u = x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}$

b) $\min_{x_1, x_2} \; p_1x_1 + p_2x_2$ subject to $u = x_1^{\frac{1}{4}}x_2^{\frac{1}{2}}$

$p_1, p_2, u > 0$.