

Mathematical Economics (2019/2020)

Exercises 1

1. Draw the indifference curve:

a) $u(x_1, x_2) = \frac{1}{2} \min\{x_1, 2x_2\}$, $u(x_1, x_2) = 8$;

b) $u(x_1, x_2) = \min\{3x_1 + x_2, x_1 + 3x_2\}$ passing through point (5, 5);

c) $u(x_1, x_2) = \min\{x_1 + 7x_2, 4x_1 + x_2\}$, $u(x_1, x_2) = 9$.

2. Solve the following utility maximization problem

a)
$$\max_{x_1, x_2} 3\left(\frac{1}{2}x_1 + 1\right)(x_2 + 2)$$

$$2x_1 + x_2 = 8$$

b)
$$\max_{x_1, x_2} x_1 + x_1x_2 + x_2 + 1$$

$$x_1 + 3x_2 = 9$$

c)
$$\max_{x_1, x_2} x_1^{0.4} x_2^{0.6}$$

$$3x_1 + 4x_2 = 5$$

d)
$$\max_{x_1, x_2} (x_1 + 1)(x_2 + 2)$$

$$p_1x_1 + p_2x_2 = I \quad p_1, p_2, I > 0.$$

3. Find the demanded bundle for a consumer whose utility function and budget constraint are the following

Utility function	Budget constraint
a) $u(x_1, x_2) = (2x_1 + 2)(x_2 + 3)$	$x_1 + 2x_2 = 6$
b) $u(x_1, x_2) = 5\left(\frac{1}{2}x_1 + 2\right)(x_2 + 4)$	$2x_1 + x_2 = 10$
c) $u(x_1, x_2) = \left(\frac{1}{3}x_1 + 3\right)(x_2 + 3)$	$3x_1 + x_2 = 30$
d) $u(x_1, x_2) = (x_1 + 2)\left(\frac{1}{4}x_2 + 2\right)$	$x_1 + 2x_2 = 20$
e) $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$	$\frac{1}{3}x_1 + 5x_2 = 3$
f) $u(x_1, x_2) = x_1^{\frac{1}{4}} x_2$	$x_1 + 2x_2 = 6$

4. Find the demand functions $x_1(p_1, p_2, I)$ and $x_2(p_2, p_1, I)$ if the budget constraint is $p_1x_1 + p_2x_2 = I$, $p_1, p_2, I > 0$ and the utility function is given by

a) $u(x_1, x_2) = \frac{1}{3}(x_1 + 2)(x_2 + 1),$

b) $u(x_1, x_2) = 2x_1^{\frac{1}{4}}x_2^{\frac{1}{2}},$

c) $u(x_1, x_2) = x_1x_2^4,$

d) $u(x_1, x_2) = a \ln x_1 + (1-a) \ln x_2, a \in (0,1),$

e) $u(x_1, x_2) = x_1^2 + x_2^2,$

f) $u(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)^4,$

g) $u(x_1, x_2) = 2 \min\{4x_1 + x_2, x_1 + 4x_2\},$

h) $u(x_1, x_2) = \min\{4x_1 + x_2, x_1 + 7x_2\}.$

5. Solve the expenditure minimization problem

a)
$$\min_{x_1, x_2} p_1x_1 + p_2x_2$$

$$u = x_1^{\frac{1}{5}}x_2^{\frac{1}{5}}$$

b)
$$\min_{x_1, x_2} p_1x_1 + p_2x_2$$

$$u = x_1^{\frac{1}{4}}x_2^{\frac{1}{2}}$$

$p_1, p_2, u > 0.$