

Mathematical Economics (2019/2020)

Exercises 3

1. Check the returns to scale for the following technologies

a) $f(x_1, x_2) = 3x_1 + x_2$, b) $f(x_1, x_2) = \sqrt{x_1 + 2x_2}$, c) $f(x_1, x_2) = x_1^{1/4} x_2^{3/4}$,
d) $f(x_1, x_2) = x_1^2 x_2^3$, e) $f(x_1, x_2) = (x_1^{1/4} + x_2^{1/4})^4$, f) $f(x_1, x_2) = \sqrt{x_1 + x_2^2}$.

2. For the following technologies

A) $y = A(ax_1^\rho + (1-a)x_2^\rho)^{\frac{1}{\rho}}$,

B) $y = A(ax_1^{-\rho} + (1-a)x_2^{-\rho})^{-\frac{1}{\rho}}$, $0 \neq \rho > -1$, $0 < a < 1$, $A > 0$

compute:

a) the marginal product of capital ($\frac{\partial y}{\partial x_1}$) and marginal product of labour ($\frac{\partial y}{\partial x_2}$),

b) the technical rate of substitution ($TRS = \frac{dx_2}{dx_1} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2}$)

c) the output elasticity of capital ($\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} \equiv \frac{\partial \ln y}{\partial \ln x_1}$) and output elasticity of labour

($\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y}$),

d) the elasticity of substitution ($\sigma = \frac{d(x_2/x_1)}{dTRS} \frac{TRS}{(x_2/x_1)} \equiv \frac{d \ln(x_2/x_1)}{d \ln(TRS)}$),

e) the elasticity of scale ($\varepsilon_t = \lim_{t \rightarrow 1} \frac{dy(tx_1, tx_2)}{dt} \frac{t}{y(tx_1, tx_2)}$),

f) $\lim_{\rho \rightarrow 0} y$.

3. A firm has a production function given by

A) $y = 4x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$; B) $y = 3x_1^{\frac{1}{4}} x_2^{\frac{1}{2}}$; C) $y = 5x_1^{\frac{1}{3}} x_2^{\frac{1}{2}}$; D) $y = 12x_1^{\frac{1}{6}} x_2^{\frac{1}{3}}$.

a) What are the factor demand functions?

b) What are the conditional factor demand functions?

c) What is the cost function?

d) What is the supply function?