

Mathematical Economics (2017/2018)

Exercises 4

1. For the following technology

$$y = A(ax_1^{-\rho} + (1-a)x_2^{-\rho})^{-\frac{1}{\rho}}, \quad 0 \neq \rho > -1, \quad 0 < a < 1, \quad A > 0$$

compute:

a) the marginal product of capital ($\frac{\partial y}{\partial x_1}$) and marginal product of labour ($\frac{\partial y}{\partial x_2}$),

b) the technical rate of substitution ($TRS = \frac{dx_2}{dx_1} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2}$)

c) the output elasticity of capital ($\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} \equiv \frac{\partial \ln y}{\partial \ln x_1}$) and output elasticity of labour

$$(\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y}),$$

d) the elasticity of substitution ($\sigma = \frac{d(x_2/x_1)}{dTRS} \frac{TRS}{(x_2/x_1)} \equiv \frac{d \ln(x_2/x_1)}{d \ln(TRS)}$),

e) the elasticity of scale ($\varepsilon_t = \lim_{t \rightarrow 1} \frac{dy(tx_1, tx_2)}{dt} \frac{t}{y(tx_1, tx_2)}$),

f) $\lim_{\rho \rightarrow 0} y$.

(1.5 p)