

Mathematical Economics (2019/2020)

Exercises 4

1. Suppose that we have two firms that face linear demand curve $p = 200 - \frac{1}{2}(y_1 + y_2)$ and their cost functions are $c_1(y_1) = \frac{1}{2}y_1^2$, $c_2(y_2) = 10y_2$, respectively.

a) Compute the Cournot equilibrium amount of output for each firm and firms' profits.

b) If firm 2 behaves as a follower and firm 1 behaves as a leader, compute the Stackelberg equilibrium amount of output for each firm and firms' profits.

Please repeat calculations if:

A) $p = 40 - \frac{1}{2}(y_1 + y_2)$; $c_1(y_1) = 2y_1^2$; $c_2(y_2) = \frac{1}{2}y_2^2$;

B) $p = 30 - 3(y_1 + y_2)$; $c_1(y_1) = 3y_1$; $c_2(y_2) = y_2$.

C) $p = 100 - 2(y_1 + y_2)$, $c_1(y_1) = 2y_1$, $c_2(y_2) = \frac{1}{4}y_2^2$.

D) $p = 40 - (y_1 + y_2)$, $c_1(y_1) = \frac{1}{4}y_1^2$, $c_2(y_2) = 3y_2$.

2. The traders' utilities are given by $u^1(x_1, x_2) = x_1x_2^2$ and $u^2(x_1, x_2) = x_1^{1/2}x_2^{1/2}$. Their initial endowments are the following $a^1 = (2, 2)$ and $a^2 = (4, 4)$. Traders come to a market and exchange commodities to maximize their utilities. Compute the price vector in equilibrium. Compare the utilities before and after the exchange.

Please repeat calculations if:

A) $a^1 = (1, 4)$, $a^2 = (2, 1)$, $u^1(x_1, x_2) = x_1^3x_2$, $u^2(x_1, x_2) = x_1^{\frac{3}{2}}x_2^{\frac{3}{4}}$.

B) $a^1 = (10, 10)$, $a^2 = (20, 5)$, $u^1(x_1, x_2) = x_1^{2/3}x_2^{1/3}$, $u^2(x_1, x_2) = x_1^{1/3}x_2^{1/2}$.

C) $a^1 = (3, 9)$, $a^2 = (1, 3)$, $u^1(x_1, x_2) = x_1x_2^{1/3}$, $u^2(x_1, x_2) = x_1^3x_2$.